SEARCH IN MACROECONOMIC MODELS OF THE LABOR MARKET

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ABSTRACT

This chapter assesses how models with search frictions have shaped our understanding of aggregate labor market outcomes in two contexts: business cycle fluctuations and long-run (trend) changes. We first consolidate data on aggregate labor market outcomes for a large set of OECD countries. We then ask how models with search improve our understanding of these data. Our results are mixed. Search models are useful for interpreting the behavior of some additional data series, but search frictions per se do not seem to improve our understanding of movements in total hours at either business cycle frequencies or in the long-run. Still, models with search seem promising as a framework for understanding how different wage setting processes affect aggregate labor market outcomes.

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In the last two decades, macroeconomists have increasingly used search theory to model the labor market. The macro-search literature is now sufficiently developed to make it meaningful to assess how integrating search theory into otherwise standard aggregate models affects the analysis of macroeconomic outcomes.\(^1\) Although search models have been used to address a wide variety of macroeconomic issues, we focus on two: short-run (business cycle) and long-run (trend) changes in aggregate labor market outcomes.

There is a hierarchy of ways in which search may be important for macroeconomic models:

1. New data: Search models draw our attention to empirical regularities and new data sets that we would typically ignore in a model without search frictions. One example is unemployment. In a search model, we can define unemployment in a manner that conforms with statistical agencies’ usage: a worker is unemployed if she is not working, available for work, and actively seeking work. Models without search can at best hope to distinguish employment from non-employment, but are silent on the distinction between unemployed and inactivity (out of the labor force). A second example is worker flows. Search models make predictions about the movement of workers between employment, unemployment, and inactivity, and between employers. They can therefore be used to understand the great variety of empirical facts about job and worker flows that economists have developed during the last two decades.\(^2\)

2. Search behavior: Search itself may play a special role in understanding the behavior of some aspects of the economy that are routinely studied in models without search, including total hours, employment, and wages. For example, employment may be low in some circumstances because employed workers are losing their jobs at a high rate. Alternatively, it may be low because unemployed workers are either not searching very intensively or are adopting very high reservation wages. Neither of these possibilities is easily explored in a model without search frictions. Search may also lead to new shocks, act to somehow amplify the effect of a given set of shocks to the economy, or, because of the slow adjustment of employment, propagate shocks through time.

3. Match-specific rents: Search models naturally give rise to match-specific rents.\(^3\) This in turn implies that, even if workers and firms exploit all the bilateral gains from

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\(^1\)We do not intend this chapter to be a comprehensive survey, but rather a critical assessment of the state of the literature. The standard treatment of search models remains Pissarides (2000). See also Mortensen and Pissarides (1999a,b) and Rogerson, Shimer and Wright (2005) for recent surveys of the labor-search literature.

\(^2\)Abowd and Kramarz (1999) and Davis and Haltiwanger (1999) reviewed these facts in the previous volume of this handbook.

\(^3\)See Manning (2010) for a discussion of other sources of match-specific rents.
trade, wages are not uniquely determined by competitive forces. This richer set of possibilities for wage setting may be important in accounting for the behavior of total hours, employment, and wages. Although search is not the only source of match-specific rents, there is something distinctive about search: rents exist at the initial meeting of the worker and the firm, and so cannot be contracted away. Other mechanisms that generate match-specific rents, such as match-specific human capital and private information, only produce rents after the match has been formed.

In this chapter, we first consolidate a variety of evidence on both short run and long changes in labor market aggregates and labor market flows in the United States and other OECD countries. We then assess the ability of search models to explain this evidence. In both contexts, it is clear that search models are useful for understanding empirical regularities in unemployment and labor market flows. Beyond this, our assessment is mixed.

In the business cycle context, we argue that the existence of search frictions does not directly improve our understanding of how labor markets function. Three long-standing issues in business cycle research concern the amplitude, persistence, and source of fluctuations in hours and employment. Consider first the question of why employment is so volatile. Search seems a promising avenue for answering this because, at least in the United States, most of the fluctuations in employment at business cycle frequencies come from workers moving between employment and unemployment. Indeed, an increase in unemployment is often seen as the hallmark of a recession, while cyclical movements in and out of the labor force are comparatively small. But we find that, relative to a frictionless version of the real business cycle model with indivisible labor (Hansen, 1985), the presence of search frictions actually moderates the optimal extent of fluctuations in employment. Intuitively, search frictions act like an adjustment cost on labor and so firms fire fewer workers during downturns to avoid costly rehiring during the subsequent boom. Because search acts as an adjustment cost, it is intuitive that it serves to increase persistence. However, we find this increase to be quantitatively unimportant. Regarding the issue of what shocks cause business cycles, search models naturally give rise to an additional source of shocks relative to frictionless models: shocks to match separation probabilities. In this view, recessions might result from shocks that cause lots of existing matches to break up. Nonetheless, we find little scope for these types of shocks to account for a large share of employment fluctuations, at least in the United States.

On the other hand, recent research suggests that there is substantial scope for search models to improve our understanding of business cycle fluctuations by providing a framework for the analysis of alternative wage determination processes. Whereas the solution to a social planner’s problem in a search framework does not seem to resolve any issues regarding
business cycle fluctuations in the labor market, decentralized search models with different wage setting rules can improve upon their frictionless counterparts. We conclude that in the business cycle context, the main substantive contribution of search models relies on the presence of match specific rents and the opportunity for a richer set of wage setting processes.

Our analysis of long-run changes in labor market outcomes similarly leads to mixed conclusions. First, while there are substantial trend changes in relative unemployment levels across countries over time, they are still small compared with the long run decrease in hours per worker and the increase in labor force participation. From this we conclude that search frictions are unlikely to be of first-order importance in understanding long-run changes in total hours of work.

However, even if changes in unemployment are not a dominant source of differences in total hours, search theory may still help us understand these changes. A key feature of the data is that countries exhibit very different flows into and out of unemployment, even when unemployment rates are the same. Search theory is useful for assessing the role of various factors that account for these differences and how they assist us in understanding why unemployment rates have changed over time.

The direct role of search in this context remains somewhat unclear. Some research attributes an important role to how workers change their search intensity and reservation wage in response to various changes in the economic environment. But similar to recent work on business cycle fluctuations, other research attributes the most important role to how wages respond to changes in the economic environment. In these models the key role of search is to give rise to match specific rents and permit a richer set of wage responses.

An outline of the chapter follows. Section 1 focuses on business cycle fluctuations. It begins by summarizing key business cycle facts regarding total hours, employment, unemployment, and worker flows. While the emphasis is on data for the United States, we also report comparable statistics for a range of OECD economies where available. We then present a benchmark business cycle model with search frictions and assess the ability of this model to account for the key facts relative to the frictionless equivalent. To make the models comparable, in both cases we focus on a social planner’s solution, which can be decentralized through a particular assumption on wage setting. We show that search frictions per se do not improve the fit between model and data. The section closes by describing recent work which considers alternative wage-setting assumptions and has been better able to account for the business cycle facts.

Section 2 focuses on long-run changes in labor market outcomes. It begins by documenting trend changes in unemployment for a large set of OECD economies. It then compares these evolutions with evolutions for total hours, participation, hours per worker, and worker
flows. We then summarize the literature that has developed to help explain the variation in long-run unemployment changes across countries, and describe in detail two recent models that feature search and provide explanations for these evolutions.

Section 3 concludes by summarizing our key findings and describing what we see as some open questions surrounding the role of search in macroeconomics. We also briefly mention more microeconomic issues in the labor market, such as the evolution of individual workers’ wages and employment, where search has proven fruitful.

1 Cyclical Fluctuations

This section explores the ability of search models to explain the behavior of labor markets at business cycle frequencies. We break our analysis into three parts, mirroring the three ways that search may be important for macroeconomic models. First we update and extend labor market facts in an earlier volume of this *Handbook* (Lilien and Hall, 1986), highlighting the connection between those facts and the structure of search-and-matching models. We argue that search models offer a useful framework for understanding data sets that we would typically ignore in a model without frictions. We then introduce a model that integrates search theory and standard business cycle theory. To keep the comparison clean, we first focus on the solution to a social planner’s problem, and so initially abstract from alternative assumptions on wage setting. We find that search itself does not resolve important puzzles in business cycle analysis, including the nature of shocks and their amplification and propagation. The final part of this section summarizes recent developments that emphasize wage rigidities in search models, a possibility that naturally arises due to the match-specific rents. It appears that such models may be important for accounting for the standard business cycle puzzles.

1.1 Facts

We begin our analysis by confirming that in the United States and most other OECD countries, most cyclical fluctuations in hours worked are accounted for by changes in the employment-population ratio. Moreover, especially in the United States, the labor force participation rate is nearly constant, so that cyclical decreases in employment are associated with roughly equal increases in unemployment. We then consider the gross inflow and outflow of workers from unemployment, showing that recessions are characterized by a sharp spike in the inflow rate and a larger and much more persistent decline in the outflow rate. We also show that the fraction of employed workers switching jobs is countercyclical, so re-
cessions, and the initial recovery from them, are characterized by an economic environment in which it is hard to find a job. This is consistent with an aggregate matching function, where the probability that a worker finds a new job is increasing in the aggregate vacancy-unemployment ratio. Finally, we document the existence and countercyclicality of the labor wedge, a wedge between the marginal rate of substitution of consumption and leisure and the marginal product of labor. All of this evidence suggests that recessions and the early stages of recoveries are periods when workers’ labor supply is constrained by the difficulty of finding a job. Search theory offers a natural framework for understanding why this may happen.

1.1.1 Hours, Employment, and Unemployment

Lilien and Hall (1986) decompose fluctuations in total hours worked into changes in employment and changes in hours worked per employed worker. They dismissed the relevance of search theory for explaining fluctuations in employment in the United States in part because “it has nothing to say about the shift of labor resources from employment to non-market activities that is an important part of the cycle.” (Lilien and Hall, 1986, p. 1032) We update their study using data from 1976Q3 to 2009Q3 for and a more comprehensive measure of hours. In contrast to the earlier chapter, we find that movements in and out of the labor force are relatively unimportant at business cycle frequencies in the United States. But in some other OECD countries, we find that fluctuations in hours per worker and movements in and out of the labor force play are an important part of changes in total hours at business cycle frequencies.

For the United States, we use a quarterly series for hours worked per person aged 16 to 64 (hereafter total hours) and for the fraction of people at work, constructed following the procedure described in Cociuba, Prescott and Ueberfeldt (2009).4 To focus on cyclical fluctuations, we detrend the data using a Hodrick-Prescott (HP) filter with the usual smoothing parameter 1600; the second part of this chapter looks at trends. Cociuba et al. extend these series back to 1959 using data that are not available online. Since the rest of our data series

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4The Bureau of Labor Statistics (BLS) constructs the underlying data series from the Current Population Survey (CPS). We first construct a monthly series for total hours, defined as the number of people at work (CPS series LNU02005053) times average hours per person at work (LNU02005054) divided by the population aged 16–64 (difference between LNU00000000 and LNU00000097), all available online since 1976Q3. We construct the monthly series for the fraction of people at work analogously. We seasonally adjust the monthly data using the Census X11 algorithm and then take quarterly averages. Following Cociuba et al., if in one month the measure of total hours is less than 96 percent of that quarter’s average, we drop the month and average the remaining two months. We need this correction because the CPS measures hours worked during a reference week, the week including the 12th day of the month. As a result, measured hours worked are low in September during years when the Labor Day holiday falls on Monday, September 7: 1981, 1987, 1992, 1998, and 2009.
Figure 1: Solid line shows total hours. Dashed line shows the fraction of people at work. Dotted line shows the fraction of people in the labor force. Gray bands indicate NBER recession dates.

The solid and dashed lines in Figure 1 show the strong comovement between detrended hours and employment. The standard deviation of detrended total hours is 1.5 percent, while the standard deviation of the fraction of people at work is 1.0 percent and the correlation between the two series is 0.96. We thus conclude that, as was the case in the earlier period, “the biggest component of the variation in hours is fluctuations in the level of employment.” (Lilien and Hall, 1986, p. 1006)

On the other hand, there is little change in the size of the labor force at business cycle frequencies, as shown by the dotted line in Figure 1. The standard deviation of the detrended labor force participation rate is 0.3 percent and the correlation with total hours is 0.67. For example, during the worst year of the 2008–2009 recession, from August 2008 until August 2009, total hours fell by 7.5 log points, the fraction of the population at work fell by 4.9 log points, while the size of the labor force fell by only 0.9 log points. Most of the

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5We measure the labor force participation rate as the number of employed people (LNU02000000) plus unemployed people (LNU03000000) divided by the population aged 16–64. We seasonally adjust and detrend the data in the same way.

6Recall from footnote 4 that hours data from September 2009 are low because of the timing of the
decline in total hours thus came from a decrease in employment, which was associated with a roughly equal increase in the unemployment rate. In contrast to the conclusions of Lilien and Hall (1986), recent data show that the size of the labor force is a secondary factor at business cycle frequencies in the United States.

Although the first part of this chapter is mainly focused on United States business cycle facts, we comment briefly on the extent to which these facts carry over to other advanced economies. Using OECD data on employment, unemployment, hours, and population in 17 countries from 1965 to 2008, we construct annual measures of total hours, hours per worker, the employment-population ratio, and the labor force participation rate.7 We detrend these series using an HP filter with parameter 100, analogous to the 1600 we use elsewhere for quarterly data.

We start by looking at the standard deviation of the cyclical component of total hours. In the United States, this is 0.018 using annual data from 1965 to 2008, while the average across 17 OECD countries is slightly larger, 0.020. Figure 2 shows that there is some variation in this measure of volatility; in particular, Finland (FI), Portugal (PT), and Spain (ES) are substantially more volatile than the United States.

Figure 3 decomposes fluctuations in total hours by showing the relative standard deviation of hours per worker and the employment-population ratio. The United States is fairly typical, with a relative standard deviation 0.6, so employment accounts for most of the volatility in total hours.8 The average across the 17 countries in our sample is even lower, 0.54, although this masks a significant amount of heterogeneity. For example, in France (FR) and Japan (JP), hours are more volatile than the employment-population ratio, and so ignoring the hours margin would seem to be a more serious omission for those countries.

Digging a bit deeper, Figure 4 plots the correlation between total hours and each of its two components, the employment-population ratio and hours per worker. The correlation

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7 Data for civilian employment, population aged 15-64, unemployment, and unemployment durations all come from the OECD Database. Data for annual hours per worker in employment are from the Groningen Growth and Development Centre (GGDC). The countries (and country codes) we use in this paper are Australia (AU), Belgium (BE, 1965–2007), Canada (CA), Denmark (DK), Finland (FI), France (FR, 1965–2007), Germany (DE), Ireland (IE), Italy (IT), Japan (JP), Norway (NO), Portugal (PT, 1970–2008), Spain (ES), Sweden (SE), Switzerland (CH), United Kingdom (UK), and United States (US). German data are for West Germany prior to 1991 and for all of Germany starting in 1991. The sample is dictated by data availability in the OECD Database for (most of) the period since 1965.

8 Note that the United States numbers we report here are not identical to the numbers we reported earlier. The time periods are different, the frequency of the data is different, and the underlying data sources are different.
Figure 2: The standard deviation of detrended total hours in log points, for 17 OECD countries from 1965 to 2008.

Figure 3: The relative standard deviation of detrended hours per worker to the detrended employment-population ratio for 17 OECD countries from 1965 to 2008.
between total hours and the employment-population ratio is 0.95 in the United States, and the correlation between total hours and hours per worker is 0.86. This same pattern does not hold in all OECD countries. While the employment-population ratio is strongly correlated with total hours everywhere, with a correlation of 0.87 on average, hours per worker is not, with an average correlation 0.48. Moreover, the correlation between the employment-population ratio and hours per worker is 0.66 in the United States, while the OECD average correlation is nearly zero, 0.05. This reinforces our earlier conclusion that fluctuations in hours per worker are not a dominant source of fluctuations in total hours at business cycle frequencies in most countries.

As previously noted, France and Japan are two prominent outliers. Not only are hours per worker more volatile than the employment-population ratio, the correlation between hours per worker and total hours is higher than the correlation between the employment-population ratio and total hours. Whether this reflects institutional features of these countries, such as legislated changes in the length of the workweek in France, remains an open question.

Finally, Figure 5 examines the cyclical component of labor force participation. Here the United States is somewhat atypical. In the United States, the correlation between labor force participation and the employment-population ratio is 0.68 and the standard deviation of labor force participation is 0.38 times the standard deviation of the employment-population
ratio. The relative standard deviation is higher in every other country except Spain. An extreme example is Switzerland (CH), where there is essentially no difference between the standard deviation of total hours (1.80 log points), the employment-population ratio (1.84), and the labor force participation rate (1.74). Moreover, the pairwise correlation between the employment-population ratio and labor force participation rate is almost perfect. For Switzerland, most of the cyclical movement in total hours is accounted for by movements between nonparticipation and employment at a fixed number of hours per worker, and so a sharp focus only on movements between unemployment and employment would be inappropriate.

### 1.1.2 Unemployment Inflows and Outflows

Starting with Blanchard and Diamond (1990), a large literature has documented distinct cyclical patterns in the gross flow of workers between employment and unemployment. We show here that recessions are typically characterized by a sharp, short-lived increase in the inflow rate of workers from employment into unemployment and a large, prolonged decline in the outflow rate of workers from unemployment into employment. Search models are useful for making sense of these empirical regularities.
We divide our analysis of gross worker flows into several pieces. To start, we focus on the flow of workers between unemployment and employment, deferring our analysis of inactivity (out of the labor force) until the next subsection. This abstraction enables us to construct measures of worker flows directly from publicly-available unemployment duration data. We later show that the main insights we develop here carry over to a framework that accounts for the large gross flows in and out of the labor force.

The motivation for our measurement of gross flows comes from search theory. The simplest version of the Mortensen and Pissarides (1994) search-and-matching model has a fixed labor force and a recursive structure in which the rate at which an unemployed worker finds a job, \( f(t) \), depends on preferences, technology, and the state of the economy, but not directly on current unemployment \( u(t) \) or employment \( e(t) \). Unemployment and employment then evolve in continuous time according to

\[
\dot{u}(t) = x(t)e(t) - f(t)u(t) = -\dot{e}(t),
\]

where \( x(t) \) is the (often exogenous) rate at which a worker exits employment for unemployment.

Our procedure for measuring the unemployment inflow rate \( x(t) \) and outflow rate \( f(t) \) follows Shimer (2007). Since actual data are available at discrete time intervals, once a month in the United States, we define the job finding probability \( F_t \) as the probability that a worker who starts month \( t \) unemployed finds a job within the month. Let \( u_{t+1}^{<1} \) be the number unemployed with duration less than one month. Then the job finding probability is

\[
F_t = 1 - \frac{u_{t+1} - u_{t+1}^{<1}}{u_t}.
\]

The term \( u_{t+1} - u_{t+1}^{<1} \) is the number unemployed for over one month in month \( t+1 \). Dividing by \( u_t \) gives the fraction of the workers who failed to find a job during month \( t, 1 - F_t \). Assuming that the job finding rate \( f(t) \) is constant during the month, Shimer (2007) proves that \( F_t \equiv 1 - e^{-f(t)} \), giving us the probability in the continuous time model that an unemployed worker finds at least one job during the month. Similarly, one can compute \( X_t \equiv 1 - e^{-x(t)} \), the probability that an employed worker loses at least one job during the month.

Using publicly available data, we can construct these series since 1948 in the United States.\(^9\) The solid line in the top panel of Figure 6 shows our series for the job finding

\(^9\)The BLS constructs the underlying data series from the CPS and seasonally adjusts it using the X12 algorithm. We use data on employment (LNS12000000), unemployment (LNS13000000), and unemployment with duration 0 to 4 weeks (LNS13008396), where the latter is our proxy for the number unemployed with duration less than one month. The redesign of the CPS instrument in 1994 significantly altered
probability, i.e., the outflow rate from unemployment. The cyclical patterns are clear: this series rises during expansions and falls sharply during recessions. For example, during the 2008–2009 recession the job finding probability fell from 41 percent to 21 percent per month. The solid line in the bottom panel shows the employment exit probability, i.e., the inflow rate into unemployment. Here one sees a sharp, short-lived spike during most recessions. During the 2008–2009 recession, it initially rose from 2.7 to 4.3 percent per month but has since fallen back to its baseline level. The general picture is one where spikes in the unemployment inflow rate drive part of the initial increase in unemployment during most downturns, but a persistently low job finding probability explains why unemployment remains high during the subsequent recovery (Fujita and Ramey, 2009).

Elsby, Hobijn and Şahin (2008) extend this methodology to construct measures of $F_t$ and $X_t$ for fourteen OECD countries. They verify that the job finding probability accounts for most of the volatility in the unemployment rate in most Anglo-Saxon countries (the United States, Canada, Australia, and New Zealand), but find an equally important role for the employment exit probability in most other OECD countries, including the United Kingdom, France, Germany, and Japan. Like Fujita and Ramey (2009), they also stress that the employment exit probability is contemporaneous with the unemployment rate, while the job finding probability lags the cycle slightly.

1.1.3 Three-State Model

There are interesting patterns in the flow of workers in and out of the labor force as well. During recessions, unemployed workers are not only less likely to find a job, but also less likely to drop out of the labor force. Employed workers are not only more likely to become unemployed, they are also less likely to drop out of the labor force. Similarly, inactive workers are more likely to become unemployed and less likely to find a job.

To show this, we measure gross worker flows in the United States using the monthly microeconomic data from the CPS. The survey is constructed as a rotating panel, with the measurement of unemployment duration (Abraham and Shimer, 2001). Prior to 1994, workers were asked their unemployment duration whenever they were unemployed. After the redesign, the unemployment duration of workers who are unemployed in consecutive months is incremented by the length of the intervening time interval. To obtain a consistent series, we use the underlying microeconomic data to construct a series for short-term unemployment for workers in the “incoming rotation groups,” i.e., workers who are asked about unemployment duration directly because they were not in the survey in the previous month. We seasonally adjust this data using the X11 algorithm and splice it with the official series that is available before 1994.

Note that because the trends are small, we show Figures 6–10 in levels rather than detrended.

The data since 1976 are available electronically from the National Bureau of Economic Research (NBER, http://www.nber.org/data/cps_basic.html).
Figure 6: The top panel shows the job finding probability and the UE transition probability. The bottom panel shows the employment exit probability and the EU transition probability. Gray bands indicate NBER recession dates.
individuals in it for four consecutive months. This means that it is theoretically possible to match up to three-quarters of the respondents between consecutive surveys, although in practice, coding errors modestly reduce the matching rate. We then measure gross worker flows between labor market states $A$ and $B$ in month $t$ as the number of individuals with employment status $A$ in month $t - 1$ and $B$ in month $t$. This yields an updated version of the gross flows data that Blanchard and Diamond (1990) analyzed.

We manipulate this data in two ways. First, consistent with our earlier analysis, we focus on the probability that a worker switches states in a given month, rather than the total number of workers switching states—i.e., transition probabilities, rather than gross worker flows. Second we adjust the data to account for time-aggregation (Shimer, 2007). To understand why this adjustment may be important, suppose an inactive worker becomes unemployed and finds a new job within a month. We would record this as an IE transition, rather than an IU and a UI transition. Similarly, a worker may reverse an EU transition within the month, and so the job loss may disappear from the gross flows entirely. Both of these events are more likely when unemployment duration is shorter.

To proceed, let $\lambda_t^{AB}$ denote the Poisson arrival rate of a shock that moves a worker from state $A \in \{E, U, I\}$ to state $B \neq A$ during month $t$, assumed to be constant within the month. Also let $n_t^{AB}(\tau)$ denote the fraction of workers who were in state $A$ at the start of month $t$ and are in state $B$ at time $t + \tau$ for $\tau \in [0, 1]$. Since $\lambda_t^{AB}$ is constant during the month, this satisfies an ordinary differential equation

$$\dot{n}_t^{AB}(\tau) = \sum_{C \neq B} n_t^{AC}(\tau) \lambda_t^{CB} - \sum_{C \neq B} n_t^{AC}(\tau) \lambda_t^{BC}. \quad (2)$$

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12 We do not adjust the data for classification error and missing observations. Abowd and Zellner (1985) and Poterba and Summers (1986) show that misclassification in one survey creates a significant number of spurious flows. For example, Poterba and Summers (1986) show that only 74 percent of individuals who are reported as unemployed during the survey reference week in an initial interview are still counted as unemployed when they are asked in a followup interview about their employment status during the original survey reference week; 10 percent are measured as employed and 16 percent are inactive. In their pioneering study of gross worker flows, Blanchard and Diamond (1990) used Abowd and Zellner’s (1985) corrected data, based on an effort by the BLS to reconcile the initial and followup interviews. Regrettably it is impossible to update this approach to the present because the BLS no longer reconciles these interviews (Frazis, Robison, Evans and Duff, 2005). Still, some corrections are possible. For example, the change in employment between months $t$ and $t + 1$ should in theory be equal to the difference between the flow into and out of employment. Fujita and Ramey (2009) adjust the raw gross worker flow data so as to minimize this discrepancy, as discussed in the unpublished working version of their paper. This does not substantially change the results we emphasize here.

13 Our analysis of the two-state model also implicitly accounted for time-aggregation. This is because $u_{t+1}^{<1}$ measures the share of unemployed workers with current duration less than one month, not the share of unemployed workers who were employed in the previous month. No such measure is available in the gross flows data.
\( n_t^{AB}(\tau) \) increases when a worker who was in state \( A \) at \( t \) and is in state \( C \) at \( t + \tau \) transitions to \( B \) and decreases when a worker who was in state \( A \) at \( t \) and is in state \( B \) at \( t + \tau \) transitions to \( C \). We solve this system of differential equations using the initial conditions \( n_t^{AA}(0) = 1 \) and \( n_t^{AB}(0) = 0 \) if \( B \neq A \). Although the expressions are too cumbersome to include here, this gives us the six independent end-of-month shares \( \{n_t^{AB}(1)\} \) as functions of the six transition rates \( \{\lambda_t^{AB}\} \), where \( A \neq B \). Moreover, we can measure the end-of-month shares \( n_t^{AB}(1) \) directly from the gross worker flow data. To recover the instantaneous transition rates \( \lambda_t^{AB} \), we invert this mapping numerically.\(^{14}\)

The dashed line in the top of Figure 6 compares the resulting three-state UE transition probability with the two-state job finding probability. The cyclical behavior of the two series is remarkably similar during the overlapping time periods, even though their levels are off by about 50 percent. The bottom panel shows that the EU transition probability likewise tracks the employment exit probability \( X_t \), although the former is noticeably more volatile than the latter. This validates the abstraction to a two-state model for the purposes of studying United States business cycles.

Figure 7 shows transitions between all three states. Our discussion focuses on the 2008–2009 recession, although similar patterns appear in most previous recessions. The top left panel again shows the EU transition probability. From the end of 2007 to the first quarter of 2009, it rose from 1.6 to 2.3 percent per month and subsequently fell back to 2.1 percent by the third quarter of 2009. On the other hand, the top right panel indicates that employed workers were less likely to drop out of the labor force during the recession; the probability declined from 2.8 to 2.4 percent per month by the end of the sample, so the overall probability of exiting employment scarcely changed. The second row shows the probability of exiting unemployment. In this case, the decline in the UE transition probability, from 30.7 percent to 18.8 percent during the 2008–2009 recession, was reinforced by a decline in the probability of dropping out of the labor force, which fell from 28.5 to 22.1 percent. Similarly, the figure shows that inactive workers were less likely to move directly into employment and more likely to move into unemployment during the recession, further increasing the unemployment rate.

There are few comparable measures of gross worker flows in other developed economies. This appears to be a data limitation. For example, Burda and Wyplosz (1994) construct a measure of flows into and out of unemployment for France, Germany, Spain, and the United Kingdom. They use data on new registrations at unemployment offices in the first three

\(^{14}\)If the eigenvalues of the discrete time Markov transition matrix are all positive, real, and distinct, the instantaneous transition rates are uniquely determined. In practice, this is the case in the United States.
Figure 7: Monthly switching probabilities from state \( A \) to state \( B \), after accounting for time aggregation \((\Lambda_{t}^{AB})\). Gray bands indicate NBER recession dates.
countries and the Labor Force Survey in the United Kingdom. Thus only the last series is comparable with the methodology we describe here. They uncover significant volatility in the number of workers entering and exiting unemployment in the United Kingdom, although the movements are uncorrelated with their preferred cyclical indicator, capacity utilization. Ponomareva and Sheen (2009) develop a four-state model for Australia by distinguishing between part-time and full-time employment. Using data since 1981, they confirm that the job finding probability falls sharply during recessions but also find sharp and persistent increases in the full-time employment to unemployment transition probability during recessions, particularly for men.

1.1.4 Employer-to-Employer Transitions

Although most job search models assume that only unemployed workers can find jobs, some newer models recognize that many workers move from employer-to-employer (EE) without experiencing an unemployment spell. Most of these papers focus on individual wage dynamics (e.g., Burdett and Mortensen, 1998; Postel-Vinay and Robin, 2002), with only a few papers examining whether EE transitions are important for understanding business cycle fluctuations (Nagypál, 2007; Moscarini and Postel-Vinay, 2008; Menzio and Shi, 2009).

Here we simply update a measure of the EE transition rate pioneered by Fallick and Fleischman (2004). Since the 1994 redesign of the CPS, respondents who are employed in consecutive months are asked “Last month, it was reported that you worked for \(x\). Do you still work for \(x\) (at your main job)?” We use the fraction of employed workers who answer this question negatively, weighted by the CPS final weights, to compute the empirical EE transition rate. A potential shortcoming of this method is that no individual is permitted to experience multiple EE movements within a month, a possibility that may be non-negligible when the job finding rate is high. A more significant issue is that the short sample period limits any analysis of the cyclical behavior of this time series.

With these caveats, Figure 8 shows this measure of the EE transition probability. Most noticeable is the secular decline in the rate, which is in part explained by the aging of the United States labor force (Shimer, 2007). When it was first constructed, about 3.5 percent of workers reported switching jobs within the month, significantly higher than the EU transition probability. That number fell to 2.5 percent during the expansion from 2002 to 2007 and fell further during the subsequent recession, reaching 1.8 percent in the third quarter of 2009, somewhat below the EU transition probability. The figure suggests that the secular decline in the EE transition probability accelerates during downturns, so employed workers are less likely to switch jobs in an adverse labor market. Again, this is consistent with the evidence.
Figure 8: The solid line shows the fraction of workers who report switching employers during the month. Gray bands indicate NBER recession dates.

from gross worker flows that it is hard to find a job during downturns.

Mazumder (2007) uses data from the Survey of Income and Program Participation (SIPP) to construct a longer time series for the employer-to-employer transition probability, from 1983 to 2003. He finds a sharp increase in the series as the United States economy emerged from the 1982–1983 recession and a trough around the 1991 and 2001 recessions. This affirms that workers switch jobs at a higher rate during booms.

We are unaware of any good time series measure of the employer-to-employer transition probability outside of the United States. Fallick and Fleischman (2004, footnote 9) discuss existing studies of the extent of employer-to-employer transitions using data from other OECD countries; however, none of these papers constructs a consistent time series.

1.1.5 Matching Function

We have argued that unemployment rises during recessions both because employed workers are more likely to lose their job and unemployed workers are less likely to find a job. Although an exact decomposition remains controversial, many studies suggest that the decline in the job finding probability is more important than the increase in the employment exit probability (Shimer, 2007; Elsby, Michaels and Solon, 2009). In any case, regardless of the
empirical evidence, search and matching models have to a large extent focused on fluctuations in the probability of finding a job. Recessions in this view are times when unemployed workers stay unemployed longer. The question is, “Why?”

Search and matching models explain fluctuations in the job finding probability through the matching function (Pissarides, 1985). The number of new matches created in month $t$, $m_t$, is a function of unemployment $u_t$ and job vacancies $v_t$, say $m_t = m(u_t, v_t)$. This implies that the job finding probability for the average unemployed worker is $F_t = m(u_t, v_t)/u_t$. A common assumption is that the matching function has constant returns to scale, so the probability that an unemployed worker finds a job is a function only of the vacancy-unemployment ratio, $F_t = f(\theta_t)$, where $\theta_t \equiv v_t/u_t$ is often called “market tightness.”

At one level, we know the matching function is an incomplete description of the job finding probability because unemployed workers are not the only ones who find jobs. We have already shown that some inactive workers move directly into unemployment and that some employed workers switch jobs without an intervening unemployment spell. It is therefore remarkable that this simple theoretical structure describes the comovement of the job finding probability and market tightness very well.

To show this, we use data on job vacancies from the Job Openings and Labor Turnover Survey (JOLTS), a monthly survey of 16,000 business establishments. According to the survey form, a job opening must satisfy three conditions: “A specific position exists; work could start within 30 days; and [the employer is] actively seeking workers from outside this location to fill the position.” The survey started in December 2000, at the peak of a business cycle, and has since followed a modest expansion and strong recession. Figure 9 shows the strong negative correlation between the unemployment rate and the vacancy rate, defined as vacancies divided by vacancies plus employment. This stable relationship is called the Beveridge curve.

Since unemployment is strongly negatively correlated with vacancies and with measures of the job finding probability, market tightness is strongly positively correlated with the job finding probability. Figure 10 shows the close link between a three month moving average of market tightness, $\theta_t = v_t/u_t$, and the quarterly series for the job finding probability. Clearly

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15 Diamond (1982) showed how increasing returns to scale in the matching process can create multiple equilibria. The subsequent literature has found scant evidence for increasing returns, however; see the survey by Petrongolo and Pissarides (2001).
16 We use BLS series JTS00000000JOL, total non-farm job openings.
17 Davis, Faberman, Haltiwanger and Rucker (2008) argue that there are significant measurement problems in the JOLTS. These mostly show up in labor turnover statistics, but they find that job openings are unreported by about eight percent, with little cyclical variation in measurement error. The BLS has since modified the reported JOLTS data to address these concerns, but in any case, this type of error would not substantially change our conclusions.
market tightness is more volatile than the job finding probability. To quantify this, suppose that the matching function is Cobb-Douglas, \( m_t = \bar{m}v_t^\eta u_t^{1-\eta} \) for some constants \( \bar{m} \) and \( \eta \). Then the job finding probability should be a constant elasticity function of market tightness, \( F_t = m_t/u_t = \bar{m}\theta_t^\eta \). We assume that there is multiplicative noise, e.g., measurement error, that disturbs this equation. Using the underlying monthly data, we then use OLS to estimate \( \eta = 0.42 \) with a standard error of 0.02.

An obvious shortcoming of the JOLTS is its brevity. Prior to 2001, the best available measure of job vacancies came from the Conference Board help-wanted advertising index. Abraham (1987) discusses this measure in detail, showing that it tracks job vacancies in regions where both series are available. Using data from 1951 to 2003, Shimer (2005b) estimates an elasticity of \( \eta = 0.28 \), somewhat smaller than the number we obtain from the shorter JOLTS data. Whether this reflects the peculiarities of help-wanted advertising, of the JOLTS data, or of the last decade remains an open question.

The survey by Petrongolo and Pissarides (2001) discusses many earlier estimates of the matching function from the United States and a variety of European countries. Most papers are interested in whether the matching function exhibits constant returns to scale and so do not constrain the coefficients on unemployment and vacancies to sum to 1. Typically they find that the coefficient on vacancies is larger than the coefficient on unemployment,
in contrast to the evidence from JOLTS and the help-wanted index, although the exact estimates differ significantly across countries. And typically they cannot reject constant returns. In any case, all of these papers establish a robust, but heterogeneous, link between unemployment, job vacancies, and the probability of finding a job. This is consistent with one of the key building blocks of search models.

1.1.6 Labor Wedge

A final fact that supports the empirical relevance of search theoretic models of the labor market is evidence that workers are constrained in their ability to supply labor during recessions. One way to express this concretely is to note that, from the perspective of a labor-market-clearing model, recessions appear to be times when there is an increase in the tax on labor. A large literature has observed this fact in United States data, noted that it is hard to observe any real movements in tax rates at these frequencies, and instead called the tax a “labor wedge” (see, for example, Parkin, 1988; Rotemberg and Woodford, 1991, 1999; Hall, 1997; Mulligan, 2002; Chari, Kehoe and McGrattan, 2007; Shimer, 2010). The existence of a counter-cyclical labor wedge is a more nuanced assertion than the other facts we document in this chapter, since it depends on some assumptions about preferences and technology. Still, it accords with many economists’ intuition that workers are not always on their labor supply curve.
We assume that a representative worker has time-separable preferences, with period utility function \( u(c, h) \) defined over consumption \( c \) and hours \( h \). We impose two restrictions on the period utility function: it must be consistent with balanced growth, so the income and substitution effects in labor supply cancel; and it must have a constant Frisch elasticity of labor supply \( \varepsilon > 0 \). Shimer (2010) and Trabandt and Uhlig (2009) show that these restrictions together impose the functional form

\[
\frac{1 - \sigma}{\sigma (1 + \varepsilon)} \left( 1 + \frac{\gamma (\sigma - 1)c h^{\frac{1+\varepsilon}{\varepsilon}}}{\sigma (1 + \varepsilon) \varepsilon} \right)^{\sigma} - 1
\]

where \( \sigma > 0 \) is a measure of the complementarity between hours worked and consumption and \( \gamma > 0 \) is the disutility of labor supply. The restriction to balanced growth preferences is quantitatively important for our results, and so we discuss it further below. In contrast, the assumption that the labor supply elasticity \( \varepsilon \) is constant is less important for our analysis. It is useful because, as we show below, \( \varepsilon \) is a key parameter for determining the magnitude of fluctuations in the labor wedge. Note that if the complementarity parameter is fixed at \( \sigma = 1 \), preferences reduce to

\[
\log c - \frac{\gamma \varepsilon}{1 + \varepsilon} h^{\frac{1+\varepsilon}{\varepsilon}},
\]

additively separable between consumption and leisure. If \( \sigma > 1 \), the marginal utility of consumption is increasing in hours worked, creating a tendency towards a positive co-movement between consumption and labor supply.

The worker faces a period budget constraint

\[
b_t = a_t + (1 - \tau_t) w_t h_t - c_t.
\]

She enters a period with some initial financial wealth \( a_t \), earns a pre-tax wage \( w_t \) per hour of work \( h_t \), pays a proportional labor tax \( \tau_t \), and consumes \( c_t \), leaving her with financial wealth \( b_t \), which is then invested in any available assets. We include time subscripts on consumption, hours, and the wage to stress that these are likely time-varying. We also include a time subscript on the labor tax because our methodology will uncover cyclical fluctuations in it. We stress that this formulation is consistent with either complete or incomplete asset markets.

The key assumption is that a worker is free to increase or decrease both her consumption and labor supply. This means she can always finance an extra \((1 - \tau_t) w_t\) units of consumption by working for an additional hour or she can reduce her consumption by this amount by working one hour less. In particular, a worker who maximizes lifetime utility subject to a
sequence of budget constraints will set the marginal rate of substitution between consumption and leisure equal to the after-tax wage. In period $t$, this gives

$$\frac{\gamma_c t^{1/\epsilon} h_t^{1/\epsilon}}{1 + \frac{\gamma (\sigma - 1) \epsilon}{\sigma (1 + \epsilon)} h_t^{1/\epsilon}} = w_t (1 - \tau_t). \quad (3)$$

Equation (3) gives a necessary condition from the worker’s optimization problem in a variety of economic environments.

We also assume that a representative firm has access to a Cobb-Douglas production technology which uses capital $k$ and labor $h$ to produce output. The firm chooses its inputs to maximize its period profits

$$A_t k_t^\alpha h_t^{1-\alpha} - r_t k_t - w_t h_t,$$

where $A_t$ is total factor productivity, $\alpha$ is the capital share of income, and $r_t$ is the rental rate on capital. Letting $y_t = A_t k_t^\alpha h_t^{1-\alpha}$ denote total output, the first order condition for the choice of labor is

$$(1 - \alpha) y_t / h_t = w_t, \quad (4)$$

which equates the marginal product of labor to the wage. Again we include time subscripts on output, hours, and the wage. Note that this holds as long as the firm is free to vary its labor at a constant wage rate $w_t$.

Now eliminate the wage between equations (3) and (4). Since the hours choice of the representative household and the representative firm are equal in equilibrium, we can write this as

$$1 - \tau_t = \left( \frac{\gamma}{1-\alpha} \right) \left( \frac{\gamma_c}{y_t} \right) h_t^{1/\epsilon} \frac{1 + \epsilon}{1 + \frac{\gamma (\sigma - 1) \epsilon}{\sigma (1 + \epsilon)} h_t^{1/\epsilon}}. \quad (5)$$

The left hand side is the proportion of labor income left after taxes. The right hand side includes several different objects: the consumption-output ratio $c_t / y_t$, hours worked $h_t$ raised to an exponent $(1 + \epsilon)/\epsilon \geq 1$, and some constants. The constants include preference parameters ($\sigma$, $\gamma$, and $\epsilon$) and a technology parameter ($\alpha$). Treating the constants as, in fact, constant at business cycle frequencies, the labor-market-clearing model predicts some co-movement between the consumption-output ratio and hours worked in response to a shock to any variable not in this equation, such as productivity or government spending.

To explore whether this relationship is a good description of the data, we use empirical measures of the consumption-output ratio and hours worked in the United States, fix

\[18\text{We measure consumption as nominal nondurable and service consumption and output as nominal GDP.}\]
Figure 11: Deviation of $1 - \tau$ from trend, $\sigma = 1$, for three different values of the Frisch labor supply elasticity $\varepsilon$. Gray bands indicate NBER recession dates.

the capital share at a conventional value of $\alpha = 1/3$, and consider different values for the complementarity parameter $\sigma$, the elasticity of labor supply $\varepsilon$, and the disutility of work $\gamma$. In all cases, we set parameters so that the average labor tax is $\tau = 0.4$.

To start, we set the complementarity parameter at $\sigma = 1$, so preferences are additively separable between consumption and leisure. This case is particularly convenient because the percent deviation of $1 - \tau_t$ from trend does not depend on the choice of the parameters $\gamma$ and $\alpha$. Rather, the consumption-output ratio is supposed to be inversely proportional to hours worked raised to the power $(1 + \varepsilon) / \varepsilon$. In the data, we find that, while there is a negative correlation between the consumption-output ratio and hours worked, hours are more volatile than $c/y$. This is inconsistent with equation (5) for any value of the labor supply elasticity $\varepsilon$. Instead, recessions look like times when workers would like to supply more labor at a wage equal to the marginal product of labor, implying that it is as if the tax on labor has increased.

Figure 11 quantifies these statements. We show three different values for the Frisch labor supply elasticity $\varepsilon$. Gray bands indicate NBER recession dates.

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19In a more complete model, this represents a combination of income taxes, payroll taxes, and consumption taxes. This is a reasonable value for the average marginal tax rate in the United States; see Prescott (2004).
supply elasticity, $\varepsilon = \frac{1}{2}$, 1, and $\infty$, corresponding to $(1 + \varepsilon)/\varepsilon = 3$, 2, and 1. In all cases, the implied value of $1 - \tau_t$ falls sharply during recessions, i.e., the labor wedge is countercyclical. The 2008–2009 recession stands out. Even with an infinite labor supply elasticity, $1 - \tau_t$ stood about 3 log points above trend at the beginning of 2008 and fell to 3 log points below trend in the latest available data, in the third quarter of 2009. Smaller labor supply elasticities exacerbate this issue.

Figure 12 examines the role of consumption-hours complementarity $\sigma$. We fix $\varepsilon = 1$ and consider three different values, $\sigma = 1$, $\sigma = 2$, and $\sigma = \infty$. Higher complementarity implies that the denominator in equation (5) is more sensitive to hours. In particular, when $\sigma > 1$, a decline in hours lowers the denominator and so raises $1 - \tau_t$, improving the fit of the model and data. But Figure 12 shows that the improved fit is modest, even in the limit with maximum curvature.

Since the labor wedge only requires data on the consumption-output ratio and hours, it is straightforward to construct a time-series measure of it in other countries. We use data on the consumption-output ratio from the Penn World Tables and on total hours from Section 1.1.1. We assume consumption and leisure are separable, $\sigma = 1$, which ensures that the disutility of work $\gamma$ and the capital share $\alpha$ do not affect the volatility of $1 - \tau_t$. Finally,
Figure 13: The relative standard deviation and the correlation of detrended $1 - \tau_t$ and detrended total hours, for 17 OECD countries from 1965 to 2004.

we fix the elasticity of labor supply at $\varepsilon = 1$.\textsuperscript{20}

Figure 13 compares the relative standard deviation of $1 - \tau_t$ and total hours with the correlation between these two series for 17 OECD countries using annual data from 1965 to 2004. Although the correlation is higher in the United States than in any other country except the United Kingdom, at 0.98, it exceeds 0.5 in every country and is below 0.75 only in Japan. Moreover, the relative volatility of the labor wedge is smaller in the United States than in most other countries. The results are reasonably robust to higher values of the labor supply elasticity. Even with $\varepsilon = \infty$, we find that the correlation between $1 - \tau_t$ and $h_t$ remains above 0.5 in every country except Japan, where it falls virtually to 0.

The bottom line is that, with these functional forms for preferences and technology, recessions look like times when the labor income tax rises. One possible interpretation is that workers are constrained from working as much as they would like during recessions, perhaps because search frictions prevent them from finding a job. Although we will explore this possibility further in the theoretical portion of this chapter, it is worth noting that with

\textsuperscript{20}The NBER working paper version of Ohanian, Raffo and Rogerson (2008) measures both the trend and the cyclical component of the labor wedge in 21 OECD countries. They assume preferences consistent with a constant Frisch elasticity of leisure (rather than labor supply) and set this equal to $-1$. Their findings are broadly similar to the ones we develop here.
other preferences, the puzzle would disappear.\footnote{Alternatively, considering richer heterogeneous agent models may be useful in accounting for the labor wedge (Chang and Kim, 2007).} For example, suppose

$$u(c, h) = c - \frac{\gamma \varepsilon}{1 + \varepsilon} h^{1+\varepsilon}.$$  

In this case, the labor wedge equation (5) becomes

$$1 - \tau = \frac{(\frac{\gamma}{1 - \alpha}) h_{t}^{1+\varepsilon}}{y_{t}}.$$  

Since output is somewhat more volatile than hours and the two outcomes are strongly positively correlated, the labor wedge is not particularly cyclical when $\varepsilon$ is sufficiently large. But we view these preferences, in particular the absence of income effects in labor supply, as implausible. They imply that at a point in time, high wage workers should supply far more labor than low wage workers. Similarly, they imply that over time, there should be a strong increasing trend in hours worked. Neither of these patterns is in the data. On the contrary, Figure 21 suggests that hours worked may be falling in the long-run. As we discuss in the second section of this chapter, part of that may be a response to rising labor and consumption taxes, but to the extent this reflects a deviation from balanced-growth preferences, it suggests that income effects are stronger than substitution effects. In this case, the results we have reported here understate the cyclicality of the labor wedge.

1.2 Theory

This section uses an explicit dynamic stochastic equilibrium search and matching model to explore whether search frictions are useful for explaining this set of business cycle facts. Our treatment here follows Shimer (2010), which in turn builds on the canonical model in Pissarides (1985) and early efforts to integrate that model into the real business cycle framework (Merz, 1995; Andolfatto, 1996).

1.2.1 Model Setup

The model examines the interaction between a representative firm and a representative household in a closed economy. The firm uses a standard production technology combining capital and labor to create a single final good, which is used both for consumption and investment. It also uses labor to recruit more workers. At the firm level, the recruiting technology is constant returns to scale, but the efficiency of recruiting is decreasing in the
aggregate recruiter-unemployment ratio.

The household has preferences over consumption and leisure that are additively separable over time and between consumption and leisure. It has many members and so can insure individuals against idiosyncratic risk, a standard device for finding a complete markets allocation. The household inelastically supplies workers to the market, although not all of them are always employed due to the search frictions.

We focus on a planner’s problem, where the planner chooses consumption, investment, and the allocation of workers to production and recruiting to maximize the household’s utility subject to the economy’s resource constraint and the search frictions. By looking at the planner’s problem, we can understand how search frictions per se affect the behavior of aggregate labor market outcomes. Section 1.3 discusses the importance of wage setting, emphasizing that “rigid wages” may arise due to match-specific rents and can significantly affect labor market outcomes.

Finally, note that our formulation abstracts from distortionary taxes, and so the labor wedge is non-zero only because search frictions create a gap between the marginal product of labor and the marginal rate of substitution between consumption and leisure.\textsuperscript{22}

**Time and States** We study a discrete time model with an infinite horizon. Denote time by $t = 0, 1, 2, \ldots$ and the state of the economy at time $t$ by $s_t$. Let $s^t = \{s_0, s_1, \ldots, s_t\}$ denote the history of the economy and $\Pi(s^t)$ denote the time-0 belief about the probability of observing an arbitrary history $s^t$ through time $t$.\textsuperscript{23} Aggregate productivity and the probability of exiting employment are both exogenous functions of history $s^t$.

**Households** A representative household has preferences over consumption $c(s^t)$ and labor supply $n(s^t)$ in history $s^t$, ordered by

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \Pi(s^t) \left( \log c(s^t) - \gamma n(s^t) \right),$$

Note that labor is indivisible and each individual suffers a utility loss $\gamma$ when employed. Thus the household effectively has an infinite Frisch elasticity of labor supply. Indivisible labor implies $n(s^t)$ is the employment rate and, normalizing the size of the household, $1 - n(s^t)$ is the unemployment rate.

\textsuperscript{22}It is straightforward to introduce a distortionary labor tax with the proceeds rebated lump-sum to households. If the planner does not internalize the tax rebate, the distortionary tax creates an additional labor wedge. This does not affect the model’s cyclical properties.

\textsuperscript{23}One might instead call the probability $\Pi_t(s^t)$ to clarify that the length of the vector $s^t$ depends on $t$. With a slight abuse of notation we simply call this $\Pi(s^t)$, and similarly for other history-dependent functions.
Recruiting  Let $\theta(s^t)$ denote the aggregate ratio of recruiters to unemployed workers in history $s^t$. Employment evolves as

$$n(s^{t+1}) = (1 - x(s^t))n(s^t) + f(\theta(s^t))(1 - n(s^t))$$  \hspace{1cm} (7)

where $s^{t+1} \equiv \{s^t, s_{t+1}\}$ is a continuation history of $s^t$. A fraction $x(s^t)$ of the employed workers lose their job and become unemployed, while the remainder stay employed. A fraction $f(\theta(s^t))$ of the $1 - n(s^t)$ unemployed workers find a job. We assume that the job finding probability $f$ is increasing in the recruiter-unemployment ratio. Conversely, each recruiter attracts $\mu(\theta(s^t))$ workers to the firm, where $\mu(\theta) \equiv f(\theta)/\theta$; this is a decreasing function. Thus at the firm level, the recruiting technology has constant returns to scale. This formulation is inspired by the empirical evidence supporting the existence of a matching function, although we model the inputs into the matching function as unemployed workers and recruiters, rather than unemployment and vacancies.24

Production  Firms have access to a standard Cobb-Douglas production technology. Total output is

$$y(s^t) = k(s^t)^\alpha(z(s^t)(n(s^t) - \theta(s^t)(1 - n(s^t))))^{1-\alpha},$$

where $k(s^t)$ is the capital stock, $z(s^t)$ is labor-augmenting productivity, and $\theta(s^t)(1 - n(s^t))$ is the number of recruiters (the recruiter-unemployment ratio times unemployment), so $n(s^t) - \theta(s^t)(1 - n(s^t))$ is the number of workers used in production, i.e., producers. Thus the economy faces a resource constraint

$$k(s^{t+1}) = k(s^t)^\alpha(z(s^t)(n(s^t) - \theta(s^t)(1 - n(s^t))))^{1-\alpha} + (1 - \delta)k(s^t) - c(s^t),$$  \hspace{1cm} (8)

where $\delta$ is the fraction of capital that depreciates in production.

1.2.2 Planner’s Problem

The planner starts history $s^t$ with capital $k$ and employment $n$. He chooses how much each individual consumes and the recruiter-unemployment ratio. Next period’s employment then follows equation (7), while next period’s capital stock satisfies the resource constraint (8). Let $V(k, n, s^t)$ denote the expected utility of the representative household when the aggregate capital stock is $k$, aggregate employment is $n$, and the history is $s^t$. Expressing the planner’s

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24 See Mortensen and Pissarides (1999a,b) for excellent surveys of search models with frictions based on the matching function.
problem recursively gives

\[ V(k, n, s^t) = \max_{c,\theta,k',n'} \log(c) - \gamma n + \beta \sum_{s^{t+1}|s^t} \frac{\Pi(s^{t+1})}{\Pi(s^t)} V(k', n', s^{t+1}), \]

where our notation assumes \( s^{t+1} \) is a continuation history of \( s^t \). The planner recognizes that next period’s employment satisfies

\[ n' = (1 - x(s^t))n + f(\theta)(1 - n) \]

and next period’s capital stock satisfies

\[ k' = k'^\alpha(z(s^t)(n - \theta(1 - n)))^{1-\alpha} + (1 - \delta)k - c. \]

To solve this, substitute the laws of motion for employment and capital into the value function and then take the first order condition for consumption:

\[ \frac{1}{c(s^t)} = \beta \sum_{s^{t+1}|s^t} \frac{\Pi(s^{t+1})}{\Pi(s^t)} V_k(k(s^{t+1}), n(s^{t+1}), s^{t+1}). \]  

The left hand side is the marginal utility of consumption, while the right hand side is the expected discounted marginal value of investment. Next take the first order condition for recruiting:

\[ (1 - \alpha)z(s^t)\kappa(s^t)^\alpha \sum_{s^{t+1}|s^t} \frac{\Pi(s^{t+1})}{\Pi(s^t)} V_k(k(s^{t+1}), n(s^{t+1}), s^{t+1}) \]

\[ = f'(\theta(s^t)) \sum_{s^{t+1}|s^t} \frac{\Pi(s^{t+1})}{\Pi(s^t)} V_n(k(s^{t+1}), n(s^{t+1}), s^{t+1}), \]

where

\[ \kappa(s^t) \equiv \frac{k(s^t)}{z(s^t)(n(s^t) - \theta(s^t)(1 - n(s^t)))} \]

is the equilibrium capital-labor ratio in the production sector. The left hand side of equation (10) is the marginal product of labor, multiplied by the value of the capital lost by a small increase in the recruiter-unemployment ratio. The right hand side is the increase in employment from the shift in the ratio, multiplied by the marginal value of employment.
Next turn to the envelope condition for capital:

\[ V_k(k(s^t), n(s^t), s^t) = (\alpha \kappa(s^t)^{\alpha - 1} + 1 - \delta) \beta \sum_{s^{t+1}|s^t} \frac{\Pi(s^{t+1})}{\Pi(s^t)} V_k(k(s^{t+1}), n(s^{t+1}), s^{t+1}). \]  

(12)

The marginal value of capital today is the marginal product of capital multiplied by the expected marginal value of capital tomorrow. Finally, the envelope condition for labor:

\[ V_n(k(s^t), n(s^t), s^t) = -\gamma 
+ (1 - \alpha) z(s^t) \kappa(s^t)^\alpha (1 + \theta(s^t)) \beta \sum_{s^{t+1}|s^t} \frac{\Pi(s^{t+1})}{\Pi(s^t)} V_k(k(s^{t+1}), n(s^{t+1}), s^{t+1}) 
+ (1 - x(s^t) - f(\theta(s^t))) \beta \sum_{s^{t+1}|s^t} \frac{\Pi(s^{t+1})}{\Pi(s^t)} V_n(k(s^{t+1}), n(s^{t+1}), s^{t+1}). \]  

(13)

The marginal value of employment is the disutility of work, plus the value of the increase in next period’s capital stock that comes from the additional output, plus the value of the increase in next period’s employment that comes having an additional worker this period.

Eliminate the expected marginal value of capital and employment from equations (12) and (13) using equations (9) and (10). This gives expressions for the current marginal value of capital and employment:

\[ V_k(k(s^t), n(s^t), s^t) = \frac{\alpha \kappa(s^t)^{\alpha - 1} + 1 - \delta}{c(s^t)}, \]

\[ V_n(k(s^t), n(s^t), s^t) = -\gamma + \frac{(1 - \alpha) z(s^t) \kappa(s^t)^\alpha}{c(s^t)} \left( 1 + \theta(s^t) + \frac{1 - x(s^t) - f(\theta(s^t))}{f'(\theta(s^t))} \right). \]

Substitute these back into the envelope conditions (12) and (13) to get

\[ 1 = \beta \sum_{s^{t+1}|s^t} \frac{\Pi(s^{t+1})c(s^t)}{\Pi(s^t)c(s^{t+1})} (\alpha \kappa(s^{t+1})^{\alpha - 1} + 1 - \delta) \]  

(14)

and

\[ (1 - \alpha) z(s^t) \kappa(s^t)^\alpha = \beta f'(\theta(s^t)) \left( -\gamma c(s^t) 
+ \sum_{s^{t+1}|s^t} \frac{\Pi(s^{t+1})c(s^t)}{\Pi(s^t)c(s^{t+1})} (1 - \alpha) z(s^{t+1}) \kappa(s^{t+1})^\alpha 
\times \left( 1 + \theta(s^{t+1}) + \frac{1 - x(s^{t+1}) - f(\theta(s^{t+1}))}{f'(\theta(s^{t+1}))} \right) \right). \]  

(15)
These equations contain the model’s main implications. Equation (14) states that the marginal cost of capital, one unit of consumption today, is equal to the expected marginal product of capital next period plus the value of the undepreciated portion of the capital, discounted using the appropriate stochastic discount factor. Equation (15) expresses the trade-off between recruiting and producing. An additional producer generates \((1 - \alpha)z(s^t)\kappa(s^t)^\alpha\) units of output in the current period. An additional recruiter yields \(f'(\theta(s^t))\) additional workers next period. Each unit of labor supplied reduces utility by \(-\gamma c(s^t)\) when measured in units of consumption. In addition, each new recruit permits the planner to put some additional workers into production, each of whom generates \((1 - \alpha)z(s^{t+1})\kappa(s^{t+1})^\alpha\) units of output. The term on the last line is the number of workers who can be placed into production in period \(t + 1\) while allowing the firm to maintain its baseline size in period \(t + 2\). This includes the recruit, some recruiters who can be shifted into production while maintaining the same recruiter-unemployment ratio \(t + 1\), and a reduction in the recruiter-unemployment ratio at \(t + 1\) enabled by the continued presence of the new recruit in period \(t + 2\).

In summary, the solution to the planner’s problem is a set of stochastic processes for consumption, capital, the recruiter-unemployment ratio, and employment that satisfies the law of motion for employment in equation (7), the resource constraint (8) and the two optimality conditions for investment in physical capital, equation (14), and the allocation of labor, equation (15).

### 1.2.3 Decentralization

It is straightforward to decentralize the planner’s problem as a search equilibrium with complete markets. A representative household chooses consumption and the purchase of Arrow securities to maximize utility subject to a lifetime budget constraint and a law of motion for employment. A representative firm purchases capital and allocates labor to production and recruiting in order to maximize the present value of profits, discounted using the intertemporal price that clears the asset market. Wages are set by Nash bargaining, where workers’ bargaining power is \(\phi\) and the threat point in bargaining is the dissolution of the match. In equilibrium, the goods and asset markets clear and the aggregate recruiter-unemployment ratio is consistent with each firm’s labor allocation decision.

Shimer (2010) shows that the conditions that define the equilibrium are nearly unchanged from the social planner’s problem. Employment still satisfies the law of motion in equation (7) and capital satisfies the resource constraint (8). The first order condition for capital in equation (14) also must hold in equilibrium. But the optimality condition for recruiting...
changes from equation (15) to
\[
(1 - \alpha)z(s^t)\kappa(s^t) = \beta\mu(\theta(s^t)) \sum_{s^t+1|s^t} \frac{\prod(s^t+1)c(s^t)}{\prod(s^t)c(s^t)} \left( (1 - \alpha)z(s^t+1)\kappa(s^t+1)^\alpha \left(1 + \frac{1 - x(s^t+1)}{\mu(\theta(s^t+1))}\right) - w(s^t+1) \right),
\]  
where \( w(s^t+1) \) is the equilibrium wage and \( \mu(\theta) = f(\theta)/\theta \) is the number of hires per recruiter. Under Nash bargaining, the wage satisfies
\[
w(s^t) = \phi(1 - \alpha)z(s^t)\kappa(s^t)^\alpha(1 + \theta(s^t)) + (1 - \phi)\gamma c(s^t).
\]

The first term on the right hand side is workers’ bargaining power times a measure of the marginal product of labor. This accounts both for the output the worker produces \( (1 - \alpha)z(s^t)\kappa(s^t)^\alpha \), and for the fact that, if bargaining fails, unemployment increases and the firm must place \( \theta(s^t) \) additional workers into recruiting in order to maintain the same recruiter-unemployment ratio. The second term is the firms’ bargaining power times the marginal rate of substitution.

One can verify that if the Mortensen (1982)–Hosios (1990) condition holds,
\[
1 - \phi = \frac{\theta f'(\theta)}{f(\theta)},
\]
then equations (16) and (17) reduce to the optimality condition for recruiting, equation (15). This is possible only if \( f(\theta) = \bar{\mu}\theta^{1-\phi} \) and so \( \mu(\theta) = \bar{\mu}\theta^{-\phi} \) for some \( \bar{\mu} > 0 \) and \( \phi \in [0, 1] \).

Under these conditions, workers’ bargaining power is equal to the elasticity of the number of matches with respect to unemployment, and similarly for firms. This ensures that each firm correctly internalizes the impact of its search on the matching possibility of other firms. Otherwise the equilibrium does not decentralize the planner’s problem. In what follows, we refer to wages that decentralize the social planner’s solution as “flexible.”

1.2.4 Calibration

We cannot solve the model explicitly, and so instead proceed numerically. We calibrate the model using facts about the United States economy, linearize it in a neighborhood of the steady state, and then describe its behavior when hit by shocks. For comparability with much of the existing business cycle literature, we focus on aggregate productivity shocks as the driving force of business cycles, but we also discuss the possible role of other shocks in this framework.
Many of the parameters are standard in the real business cycle literature, but some are specific to the matching model. We start with the more familiar parameters. We think of a time period as one month so to be able to capture the typical short duration of an unemployment spell. The discount factor is $\beta = 0.996$, just under five percent annually.

We fix $\alpha = 0.33$ to match the capital share of income in the National Income and Product Accounts. We then set set $\delta = 0.0028$ per month, which pins down the capital-output ratio at 3.2 along a balanced growth path. This is the average capital-output ratio in the United States since 1948.\(^{25}\)

We assume productivity has a deterministic trend, $\log z(s^t) = \bar{s}t + s_t$, where $\bar{s}$ is mean productivity growth and $s_t$ follows a first-order autoregressive process,

$$s_{t+1} = \rho s_t + \varsigma v_{t+1}, \quad (18)$$

where $v_{t+1}$ is a white noise innovation with mean zero and standard deviation 1. Mean productivity growth is $\bar{s} = 0.0018$, about 2.2 percent per year, consistent with the annual measures of multifactor productivity growth in the private business sector constructed by the Bureau of Labor Statistics.\(^{26}\) We set the autocorrelation of productivity growth to $\rho = 0.98$ and the standard deviation to $\varsigma = 0.008$. These values are similar to standard calibrations of total factor productivity (Cooley and Prescott, 1995), with an adjustment to account for the fact that time periods are one month long.

We turn next to the parameters that determine flows between employment and unemployment. Shimer (2005b) measures the average exit probability from employment to unemployment in the United States at $x = 0.034$ per month, and we stick with that number here. Initially we assume that it is constant, but we also develop a version of the model with shocks to the employment exit probability.

Although there are many estimates of the matching function $f$ in the literature (see the survey by Petrongolo and Pissarides, 2001), most papers assume that firms create job vacancies in order to attract unemployed workers and so estimate matching functions using data on unemployment and vacancies. The technology in this paper is slightly different, with firms using workers to recruit workers. Unfortunately we are unaware of any time series showing the number of workers (or hours of work) devoted to recruiting, and so the choice of $f$ is somewhat arbitrary. Still, following much of the search and matching literature,

\(^{25}\)More precisely, we use the Bureau of Economic Analysis’s Fixed Asset Table 1.1, line 1 to measure the current cost net stock of fixed assets and consumer durable goods. We use National Income and Product Accounts Table 1.1.5, line 1 to measure nominal Gross Domestic Product.

\(^{26}\)See ftp://ftp.bls.gov/pub/special.requests/opt/mp/prod3.mphtablehis.zip, Table 4. Between 1948 and 2007, productivity grew by 0.818 log points, or approximately 0.014 log points per year. Our model assumes labor-augmenting technical progress, and so we must multiply $\bar{s}$ by $1 - \alpha$ to obtain TFP growth.
we focus on an isoelastic function, \( f(\theta) = \bar{\mu} \theta^n \), and look at the symmetric case, \( \eta = 0.5 \). We discuss below the importance of this parameter. To pin down the efficiency parameter in the matching function \( \bar{\mu} \), we build on evidence in Hagedorn and Manovskii (2008) and Silva and Toledo (2009). Those papers argue that recruiting a worker uses approximately 4 percent of one worker’s quarterly wage, i.e., a recruiter can attract approximately 25 new workers in a quarter, or 8.33 in a month. We use this fact and data on the average unemployment rate to determine \( \bar{\mu} \). We proceed in several steps. First, from (7), the steady state employment rate satisfies

\[
\frac{f(\theta)}{x + f(\theta)}.
\]

Setting \( n = 0.95 \), the average share of the labor force employed during the post-war period, and \( x = 0.034 \), this implies \( f(\theta) = 0.646 \) in steady state. Second, the functional form \( f(\theta) = \bar{\mu} \theta^n \) implies

\[
\bar{\mu} = \frac{f(\theta)}{\theta^n} = f(\theta)^{1-\eta} \mu(\theta)^n,
\]

where the second equation follows because \( \mu(\theta) \equiv f(\theta)/\theta \). From this equation, we set \( \bar{\mu} = 2.32 \), consistent with \( f(\theta) = 0.646 \), \( \mu(\theta) = 8.33 \), and \( \eta = 1/2 \). Note that this implies that the recruiter-unemployment ratio is \( \theta = f(\theta)/\mu(\theta) \approx 0.078 \). It follows that the share of recruiters in employment is \( \theta(1-n)/n \approx 0.004 \), with 99.6 percent of employees devoted to production. Thus in this calibration, the implicit hiring costs are small, at least on average.

Finally, we set the parameter governing the taste for leisure to obtain a five percent unemployment rate along the balanced growth path; this implies \( \gamma \approx 0.785 \). When we consider alternative calibrations, we vary \( \gamma \) to ensure that the unemployment rate is unchanged.

1.2.5 Results

The search model is useful for developing a notion of unemployment and a theory of worker flows between employment and unemployment. But this section asks whether the model helps to explain other shortcomings of standard business cycle models. What shocks hit the economy? How are they amplified and propagated through time? Why do they create a countercyclical labor wedge?

Shocks We focus in this section on two sources of shocks. One is completely standard in the real business cycle literature (Kydland and Prescott, 1982), the productivity shock introduced above. The other is special to frictional markets, a shock to the probability of exiting employment. This arguably resembles a “sectoral shift,” with many workers losing their job and enduring an unemployment spell before moving elsewhere (Lilien, 1982). Following Blanchard and Diamond (1989), we label the first shock “aggregate” and the second
“reallocational,” but obviously the names are only suggestive. One important question is whether quantitatively reasonable reallocation shocks are important for the dynamics of employment. We could symmetrically consider a shock to the matching function. In our view, it is implausible to argue that unemployment falls during an expansion because the matching process has exogenously improved, and so we take these shocks off the table.

In principle, one could introduce other shocks to the model. For example, it is straightforward to modify the government budget constraint and resource constraint to introduce stochastic government spending. Trigari (2009) develops a version of a search and matching model with nominal rigidities; in such a framework, monetary policy shocks can also have real effects on output. Of course, one could study both of these shocks in a model without search frictions, and our intuition is that our results comparing models with and without search frictions carry over to these shocks.

**Amplification** One of our main results is that search frictions dampen the effect of productivity shocks. That is, we compare the volatility of employment and output in a model with search frictions to one without search frictions, i.e., where firms can costlessly adjust employment and wages clear the labor market. We maintain the assumption that leisure is indivisible and so preferences are given by equation (6). This is therefore essentially the Hansen (1985) model. The first row in Table 1 considers this frictionless model. The first three columns show the theoretical, infinite sample standard deviation of output, employment, and the consumption-output ratio. The last three columns show a measure more comparable to empirical estimates of these objects. We simulate 402 months of data from our model, compute quarterly averages and then detrend using an HP filter with smoothing parameter 1600. We show the average results from 1000 such simulations of the model. In both cases output is about 2.2 times as volatile as total factor productivity, while employment and the consumption-output ratio are slightly less volatile than output. Note that in each case the standard deviation of employment and the consumption-output ratio is the same. Indeed, we know from equation (5) that, since $\varepsilon = \infty$ and $\sigma = 1$, employment and the consumption-output ratio mirror each other.

The second row shows our baseline search model. We feed the same shock into the model and affirm that search frictions dampen the response. The standard deviation of detrended

---

27 Merz (1995) and Andolfatto (1996) assume that recruiting costs are in units of goods rather than labor. In that case, search frictions do not substantially dampen the response to productivity shocks. We view a time-intensive model of recruiting as more plausible.

28 Employment and the consumption-output ratio are less persistent than productivity, while output is about equally persistent. Therefore detrending boosts the volatility of $n$ and $c/y$ relative to productivity but does not much affect the volatility of $y$ relative to productivity.
output falls by 51 percent, the standard deviation of the detrended consumption-output ratio by 54 percent, and the standard deviation of detrended employment falls most of all, by 90 percent. The theoretical standard deviations fall by a similar magnitude. To the extent that one hoped search frictions would amplify productivity shocks, the results are disappointing. Intuitively, increasing the recruiter-unemployment ratio in response to a positive productivity shock is costly because doing so reduces the effectiveness of each recruiter. This naturally dampens the volatility of employment and hence output. Shimer (2010) verifies that if search frictions are more severe, so each recruiter attracts fewer workers per month, the dampening effect of frictions is even more extreme.

We next introduce the reallocation shock. We assume \( \log x(s^t) = \log \bar{x} + s_{x,t} \), where \( \bar{x} = 0.034 \) and \( s_{x,t} \) follows a linear process,

\[
s_{x,t+1} = \rho_x s_{x,t} + \varsigma_x v_{x,t+1},
\]

We assume the innovation to reallocation, \( v_{x,t+1} \), is white noise with mean 0 and standard deviation 1. We fix \( \rho_x = 0.83 \) and \( \varsigma_x = 0.034 \) so as to match the autocorrelation of the employment exit probability and its unconditional standard deviation, as measured in Shimer (2007). For simplicity, we assume that the productivity shock and the employment exit probability shock are uncorrelated, so

\[
s_{z,t+1} = \rho_z s_{z,t} + \varsigma_z v_{z,t+1},
\]

where \( v_{z,t+1} \) is independent white noise with mean 0 and standard deviation 1. We then leave \( \rho_z = 0.98 \) and \( \varsigma_z = 0.008 \), as in the model without search frictions.\(^{29}\)

We can also introduce correlation between the shocks, for example by making productivity \( s_{z,t+1} \) a function of both the productivity shock \( v_{z,t+1} \) and the reallocation shock \( v_{x,t+1} \). One way to set the correlation

---

**Table 1**: Relative standard deviation of output \( y \), employment \( n \), and the consumption-output ratio \( c/y \) in four models. All variables are expressed relative to the standard deviation of total factor productivity \( z(s^t)^{1-\alpha} \). The first three columns show the theoretical, infinite sample standard deviations of monthly variables. The last three columns show detrended quarterly averages based on 402 months of data.

<table>
<thead>
<tr>
<th>Model</th>
<th>Theoretical</th>
<th>Finite Sample</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( y )</td>
<td>( n )</td>
</tr>
<tr>
<td>frictionless</td>
<td>2.18</td>
<td>1.39</td>
</tr>
<tr>
<td>baseline search</td>
<td>1.37</td>
<td>0.15</td>
</tr>
<tr>
<td>reallocation shocks</td>
<td>1.37</td>
<td>0.18</td>
</tr>
<tr>
<td>training cost</td>
<td>2.11</td>
<td>1.30</td>
</tr>
</tbody>
</table>

\(^{29}\)We can also introduce correlation between the shocks, for example by making productivity \( s_{z,t+1} \) a function of both the productivity shock \( v_{z,t+1} \) and the reallocation shock \( v_{x,t+1} \). One way to set the correlation
in Table 1 shows that introducing these shocks barely affects the volatility of output or the consumption-output ratio. It raises the theoretical standard deviation of employment by 20 percent and the standard deviation of detrended employment by 50 percent. Nevertheless, employment remains far less volatile than in the frictionless model.

Our calibration strategy tightly pins down most of the important model parameters. The one exception is the elasticity of the matching function, \( \eta \). Recall that increasing the recruiter-unemployment ratio in response to a positive productivity shock is costly because doing so reduces the effectiveness of each recruiter. The parameter \( \eta \) governs how quickly recruiters’ effectiveness falls. When we recalibrate the model with a lower elasticity \( \eta \), corresponding to the case where unemployed workers are a more important part of the search process, the volatility of each of the three outcomes is significantly dampened. At the extreme case of \( \eta = 0 \), \( f(\theta) = \bar{\mu} \) and so the job finding probability is constant. Absent fluctuations in the fraction of employed workers who become unemployed, \( x \), employment is constant as well, \( n = \bar{\mu}/(x + \bar{\mu}) \). The search model is equivalent to real business cycle model with inelastic labor supply.

Conversely, when we raise \( \eta \), volatility increases. An extreme case is \( \eta = 1 \), so \( f(\theta) = \bar{\mu}\theta \). This implies that a firm must shift \( 1/\bar{\mu} \) workers away from production in order to hire one new employee. It may be natural to interpret this parameterization of the model as a training cost, rather than a recruiting cost, since the cost depends only on the number of workers hired and not on the availability of unemployed workers. The last row in Table 1 shows that the model with training costs generates almost the same volatility as the frictionless model. Indeed, the volatility of detrended output and the detrended consumption-output ratio actually increase.\(^{30}\) This result accords with the finding in Mortensen and Nagypál (2007), that a model with training costs generates more volatility than a model with search frictions (see also Pissarides, 2009). Still, the point remains that training costs slightly reduce the volatility of employment compared to a frictionless model, and so in this sense they are a step in the wrong direction.

Finally, it seems worth comparing the predictions of this model with the Lucas and Prescott (1974) search model (see also Alvarez and Veracierto, 1999, 2001). That model focuses on the time-consuming reallocation of workers across labor markets that are continually hit with idiosyncratic productivity shocks. Moving to a new labor market takes

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\(^{30}\) These results appear to be a consequence of time aggregation. When we work directly in a model with a quarterly time period, the standard deviation of all three detrended variables is slightly smaller in the training cost model than in the frictionless model. In any case, the difference in volatility between the two models is small.
one period. This is equivalent to assuming that only unemployed workers are useful in the
matching process, \( f(\theta) = \bar{\mu} \), and so our analysis here would suggest a dampened response
to productivity shocks. Indeed, employment changes in that model only to the extent that
the inflow rate into unemployment changes, which we have seen empirically accounts for
only a minority of aggregate unemployment fluctuations.\(^{31}\) Veracierto (2008) attempts to
address this by allowing workers to drop out of the labor force, but when he does this, he
finds that labor force participation becomes so strongly procyclical in the model that the
unemployment rate is weakly procyclical.

**Propagation**  Although search frictions do not amplify shocks, they do affect how they are
propagated through time by slowing down the adjustment of employment. To see this, we
focus on the first and second order autocorrelation of output growth and employment growth.
In United States data, such growth rates are all positively serially correlated: the first and
second order autocorrelation of detrended quarterly output growth are 0.23 and 0.08, while
the corresponding numbers for detrended quarterly employment growth are 0.19 and 0.17.\(^{32}\)
This recalls the findings of Cogley and Nason (1995), who emphasize the importance of
autocorrelations for model evaluation.

Table 2 therefore shows corresponding numbers for the model. The first column shows
the theoretical correlation between the growth rate of output from month \( t \) to \( t + 3 \) and its
growth rate from \( t + 3 \) to \( t + 6 \). The second column shows similar numbers for employment.
We see that in the frictionless model, both of these correlations are negative. This reflects
the fact that shocks are mean reverting and so output and employment also tend to revert
to trend. Adding search frictions boosts this correlation, particularly for employment. On
the other hand, reallocation shocks lower the autocorrelation of employment by buffeting
it with relatively transitory shocks. Curiously, the training cost model, where matching
depends only on recruiters, significantly raises the theoretical autocorrelation of output.
These columns suggest that search frictions, particularly the training cost variant, raise the
persistence of output and employment.

A direct comparison of the data with the numbers in the first two columns is diffi-
cult, both because the data are detrended from a finite sample and because the data are
time-aggregated. The third and fourth columns in Table 2 therefore show the first order au-

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\(^{31}\)Strictly speaking, this is not correct. In the original Lucas and Prescott (1974) model, search is directed,
but it takes one period to arrive in the desired labor market. During that period, labor market conditions
may worsen substantially, inducing the worker to refuse the job. In practice, this event is rare and so in
most cases unemployment lasts for one period.

\(^{32}\)The output numbers are for real GDP from the National Income and Product Accounts, Table 1.1.6,
from 1976Q3 to 2009Q3. The employment numbers are for the fraction of the population who is at work.
Finite Sample

First Order

Second Order

Table 2: Autocorrelation of quarterly output and employment growth in four models. The first two columns show the theoretical, infinite sample autocorrelation of output and employment growth at quarterly frequencies. The next two columns show the first order autocorrelation of detrended quarterly output and employment growth based on 402 months of data. The last two columns show the second order autocorrelation of detrended quarterly output and employment growth based on 402 months of data.

tocorrelation from similarly time-aggregated model-generated data. Our first result is that, even in the frictionless model, output and employment growth are both positively autocorrelation (0.11). This turns out to be due entirely to time-aggregation. Indeed, even though productivity is mean reverting, the first order autocorrelation of productivity growth is also 0.11. This is because time aggregation raises the first order autocorrelation of the growth rate of an autoregressive process; see, for example, Working (1960) for the case of a random walk. This suggests it may be difficult to compare the model with data. Nevertheless, model versus model comparisons are instructive. We again see in the remaining entries in the third and fourth columns that search frictions boost the first order autocorrelation of output and employment growth, particularly in the training cost model, while reallocation shocks moderate the autocorrelation of employment.

Finally, the last two columns in Table 2 look at second order autocorrelations. For a time-aggregated random walk, the first order autocorrelation of growth rates is positive, but the second order autocorrelation is always zero. Therefore we expect that looking at the second order autocorrelation will moderate any issues related to time aggregation. Indeed, we find that the second order autocorrelation of output and employment growth is consistently negative and is basically unaffected by the presence of search costs. None of these search models can therefore generate the persistent positive autocorrelation of output and employment growth that we observe in United States data.

Labor Wedge Finally, we turn to the labor wedge. We imagine an economist who understands that the labor supply elasticity is $\varepsilon = \infty$ and the consumption-hours complementarity is $\sigma = 1$. He measures the labor wedge $\tau$ using equation (5). Table 3 shows the standard
deviation of measured $1 - \tau$ and its comovement with output and employment.\footnote{Note that the choice of the disutility of work $\gamma$ does not affect the statistical properties of $1 - \tau$, although it determines its average level.}

In the frictionless model, the labor wedge is always equal to 0, while in the search model it is volatile, and more volatile in the baseline model than the one with training costs. The problem is that $1 - \tau$ is negatively correlated with output and employment in the model, the opposite of the data. This is intuitive. The labor wedge ignores the existence of search frictions, which act as an adjustment cost. Relative to a frictionless model, the adjustment cost dampens fluctuations in employment (Table 1). To rationalize this in a frictionless model, we need to assume that the tax on labor rises during every expansion. But this is exactly the opposite of what we observe in the data. Again, the negative correlation between the labor wedge and output or employment is smaller in the training cost version of the model, but the counterfactual implication remains highly significant.

**Other Moments** One can examine other moments in the model, for example the behavior of consumption and investment. Doing so reaffirms earlier work emphasizing that the presence of search frictions does not substantially modify the behavior of a business cycle model (Merz, 1995; Andolfatto, 1996). The main effect of search frictions is instead to dampen the response of the economy, and especially the labor market, to aggregate shocks. While this is disappointing, it is worth stressing that the model has some advantages over the baseline business cycle model, namely the introduction of unemployment and aggregate worker flows.

<table>
<thead>
<tr>
<th></th>
<th>Theoretical</th>
<th>Finite Sample</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>std. dev.</td>
<td>correl. with</td>
</tr>
<tr>
<td></td>
<td>$y$</td>
<td>$n$</td>
</tr>
<tr>
<td>frictionless</td>
<td>0</td>
<td>—</td>
</tr>
<tr>
<td>baseline search</td>
<td>0.56</td>
<td>-0.73 -0.96</td>
</tr>
<tr>
<td>reallocation shocks</td>
<td>0.38</td>
<td>-0.72 -0.77</td>
</tr>
<tr>
<td>training costs</td>
<td>0.06</td>
<td>-0.46 -0.40</td>
</tr>
</tbody>
</table>

Table 3: Standard deviation of $1 - \tau$ and correlation with output and employment in four models. The first three columns show the theoretical, infinite sample standard deviation of $1 - \tau$ and correlation with output and employment. The last three columns show the standard deviation of the detrended quarterly labor wedge and its correlation with detrended output and detrended employment based on 402 months of data.
1.3 Rigid Wages

The concern that search models do not generate substantial fluctuations in unemployment was first voiced in Shimer (2005b), albeit in a model with linear utility and no capital; see also Costain and Reiter (2008). The body of Shimer’s paper focused on a model with Nash bargaining that satisfied the Mortensen-Hosios condition, i.e., what we have defined as a flexible wage model with outcomes equivalent to the social planner’s problem. While wages that decentralize the social planner’s solution may be a useful benchmark, the assumption is not obviously more plausible than a myriad of possible alternatives.\textsuperscript{34} In his conclusion, Shimer argued that wage rigidities—wages that are less procyclical than those which decentralize the social planner’s solution—may help to resolve the “unemployment volatility puzzle.”\textsuperscript{35} This section starts by reviewing the subsequent theoretical literature on wage rigidities, then discusses papers that attempt to measure whether wages are rigid in reality, and concludes by considering whether one needs a model with search frictions to analyze wage rigidities.

1.3.1 Theory

Hall (2005) was the first paper to quantify the possibility of wage rigidities creating volatile unemployment in a search model. He replaced the Nash bargaining assumption, analogous to equation (17) here, with a restriction that wages do not move in response to aggregate productivity shocks. A temporary increase in productivity therefore raises the revenue from hiring workers without raising the cost. This induces firms to recruit more workers, which in turn lowers the unemployment rate. Hall (2005) established that this is indeed a powerful amplification mechanism.

An important insight of Hall (2005) was that the wage negotiation between a matched worker and firm is a zero-sum game. The marginal rate of substitution between consumption and leisure is strictly less than the marginal product of labor in a search equilibrium, with the difference representing a match-specific rent due to the existence of the search friction.

\textsuperscript{\textsuperscript{34}}A number of papers assume that firms post wages offers and workers can direct their search towards their preferred offer, as in the competitive search literature (Montgomery, 1991; Peters, 1991; Moen, 1997; Shimer, 1996; Acemoglu and Shimer, 1999; Burdett, Shi and Wright, 2001; Mortensen and Wright, 2002). The equilibrium of that model coincides with the social planner’s solution, and so wages are flexible. Models of wage rigidities therefore typically assume either that firms cannot commit to wages or that workers cannot direct their search.

\textsuperscript{\textsuperscript{35}}This recognition of the central role of wage determination in search models is not new. In his review of the first edition of Pissarides (2000), Mortensen (1992, p. 166) noted that “the fact that alternative rules of wage determination may have different implications is an important neglected topic…. Unlike the Walrasian theory, there is no unique concept of equilibrium price inherent in the theory of markets with transactions costs. Wages must be determined by some form of bargaining and the implications of the model are generally sensitive to which bargaining solution is imposed.” Caballero and Hammour (1996) also stress that how match-specific rents are appropriated may be important for business cycle fluctuations.
It follows that there is a range of wages that a worker is willing to accept and a firm is willing to pay. This has two implications. First, search models with rigid wages do not suffer from the Barro (1977) critique of unemployment in the implicit contracts literature (e.g. Baily, 1974; Gordon, 1974; Azariadis, 1975), that inefficient layoffs arise only because matched workers and firms fail to exploit some of the bilateral gains from trade.\textsuperscript{36} Second, wage rigidities in existing employment relationships are inconsequential, so long as they do not lead to inefficient separations (Shimer, 2004).\textsuperscript{37} Instead, the recent search literature has focused on how wage rigidities affect firms' incentive to create job vacancies and to recruit new employees, leading to fluctuations in the job finding probability. Given the empirical evidence that the job finding probability declines sharply and remains low long after the initial recessionary shock, this emphasis seems reasonable.

The subsequent theoretical literature on wage rigidities in search models can largely be divided along two dimensions. First, some papers attempt to provide a deep foundation for the rigidity, while others pursue a more ad hoc approach. Second, in some papers the wage rigidity is intrinsically static, while in others it introduces an additional state variable.\textsuperscript{38}

The simplest wage rigidity models are ad hoc and static. One example is Hall (2005), who assumed wages are fixed forever. Hall presents a simple bargaining game in which any wage between the marginal rate of substitution and the marginal product of labor is an equilibrium. He then argues that “a constant wage rule may be interpreted as a wage norm or social consensus.” (p. 56) Blanchard and Galí (2008) also impose an ad hoc, static wage rule. Generalizing Hall (2005), they assume that the wage is proportional to productivity, but the constant of proportionality is smaller than 1. Thus when productivity is high, the gap between the marginal product of labor and the wage is large, again encouraging firms to hire. The free proportionality parameter affects the extent of wage rigidity and hence the volatility of unemployment. It is worth stressing that even with this free parameter, the model is testable. For example, Hall (2009) examines the implications of a rigid wage model for the behavior of hours per worker, under the assumption that workers and firms negotiate hours efficiently, even if total compensation is rigid. He finds the model can eliminate the cyclicality of the labor wedge.

\textsuperscript{36}With two-sided asymmetric information, inefficient separations may be a necessary feature of equilibrium. See Ramey and Watson (1997) for an example where layoffs arise in a search model because of endogenous limits on contracting.

\textsuperscript{37}With incomplete markets, wage rigidities in existing employment relationships matter because they affect the value of the relationship and so affect job creation; however, Rudanko (2009) finds that this effect is quantitatively small.

\textsuperscript{38}There may also be institutional reasons, such as unions, why wages are rigid. We do not know of any recent attempts to understand whether wage rigidities at business cycle frequencies are consistent with unions' objective function.
Other authors present more sophisticated arguments for why wages are rigid. An early example is Hagedorn and Manovskii (2008), although it is worth noting that the authors do not interpret their paper as one with wage rigidities. They calibrate the Nash bargaining parameter using information that wages move less than one-for-one with productivity, which gives them a small value for the workers’ bargaining power $\phi$ in (17). This significantly amplifies productivity shocks relative to the baseline search model. To understand why, recall that if $\phi = 1 - \eta$, so that the Mortensen (1982)–Hosios (1990) condition is satisfied, the equilibrium is equivalent to the solution to the social planner’s problem. Also recall that we also found that when $\eta = 1$, i.e., the training cost model, search frictions do not much dampen productivity shocks. It turns out that when $\phi = 0$ but $\eta < 1$, the results are similar although not quite as strong; search frictions still dampen productivity shocks, but not as much as in the baseline search model. For example, if $\phi = 0$ and $\eta = 0.5$, the standard deviation of detrended employment is 1.3 times the size of the productivity shock, far more than in the baseline search model but somewhat less than the value of 1.7 in the training cost model. The autocorrelation of output and employment growth are also essentially the same as in the training cost model and the procyclical labor wedge is unaffected as well.

Another approach to static wage rigidities comes from reexamining the threat point when bargaining. In equation (17), we assumed that a breakdown in bargaining led to the dissolution of a match. Hall and Milgrom (2008) argue that a worker and firm are likely to continue bargaining even if agreement is not immediate. Therefore the threat point when bargaining is delay, not breakdown. This small change has a big effect on the equilibrium, since the value of delay—say the worker’s time and the firm’s foregone production—is less cyclical than the value of dissolution. Such a model can potentially generate more volatility than the frictionless benchmark and a countercyclical labor wedge.

A third approach to static wage rigidities is based on asymmetric information. Kennan (2010) explores what happens if workers are unable to observe the productivity of their match. He shows that under some conditions the information rent accruing to firms is procyclical, effectively generating rigid wages and amplifying the impact of productivity shocks. Other information frictions, such as the need to pay a high wage that keeps workers from shirking (Shapiro and Stiglitz, 1984; MacLeod and Malcomson, 1989) may also be important for wage setting; see Costain and Jansen (2009) for recent work integrating efficiency wages into a search model.

These static stories amplify shocks but typically do not propagate them. Models where wages are backward looking can do both. Again, the simplest models are ad hoc. In Blanchard and Galí (2007), the current wage is a weighted average of the previous period’s wage and the marginal rate of substitution between consumption and leisure. In Shimer (2010,
Chapter 4), it is a weighted average of the previous wage and the current wage that would prevail if there were Nash bargaining, which significantly propagates shocks without much affecting the comovement of wages and labor productivity.

Gertler and Trigari (2009) provide a deeper theory for why wages may be backward looking. They assume that workers and firms only periodically negotiate, bargaining so as to satisfy the Nash solution and fixing the wage until the next opportunity to renegotiate. Crucially the negotiated wage applies not only to the firm’s existing workers, but also to any new workers it might hire. Thus firms that last negotiated their wage prior to an adverse productivity shock will have little incentive to recruit new workers following the shock. They again show that this amplifies the effect of shocks on the labor market with little consequence for other macroeconomic outcomes.

1.3.2 Evidence

This theoretical literature points to an obvious empirical question: are wages in reality rigid? Some recent papers have in fact argued that wages in new matches are flexible, as evidenced by the fact that they are as volatile as labor productivity (Pissarides, 2009; Haefke, Sonntag and van Rens, 2008); however, this evidence is also consistent with a rigid wage model. To understand why, it is useful to step back and think about an otherwise frictionless model where, for some reason, wages are above the market-clearing level. In this case, firms set the level of employment so that the marginal product of labor is equal to the wage. With a Cobb-Douglas production function, it follows that labor productivity, i.e., the average product of labor, is proportional to the wage; see equation (4).

\[ \frac{w_t h_t}{y_t} \]

This is true regardless of the source of shocks, and indeed regardless of whether wages are rigid. That is, in the absence of search frictions, the observation that wages are as volatile as labor productivity is uninformative about whether wages are rigid. This argument does not exactly carry over to a model with search frictions, but quantitatively it is not far off. Shimer (2010, Chapter 4) reports that the labor share, \( \frac{w_t h_t}{y_t} \), is nearly constant in a model where wages are extremely backward looking. Devising appropriate tests for whether wages are rigid remains an important issue for future research.

\[ \text{For evidence in support of this theory, see Galí, Gertler and López-Salido (2007), who attribute all the volatility in the labor wedge to the “wage markup,” the ratio of the real wage to the marginal rate of substitution in equation (3). They find almost no volatility in the “price markup,” the ratio of the marginal product of labor to the real wage in equation (4).} \]
1.3.3 Why Search?

A final issue is whether search models are the right framework for thinking about wage rigidities. It is clear that search frictions provide one possible explanation for why wages are rigid: there is a gap between the wage that workers will accept and firms will pay, and any wage between those bounds is in some sense reasonable. But anything that creates match-specific rents, such as training costs, match-specific capital, labor unions, and collusion among employers, also creates rents.\(^{40}\)

A natural question is therefore whether there is any reason to prefer search over other models of rents. An important feature of search models is that it is impossible for a worker and a firm to contract on the division of rents before the rents are created, since they have not yet met. In contrast, it is in principle possible to write contracts that divide the ex post rents from training costs, match-specific capital accumulation, or private information. Of course, there may be limits to that contracting. For example, limits on workers’ ability to commit to stay in an employment relationship may mean that a firm will only invest in a worker’s human capital if the worker can post a bond; and borrowing constraints may prevent bond-posting. Alternatively, employees’ morale may constrain the firm and prevent it from cutting wages during downturns (Bewley, 2002). Still, search offers a potentially important explanation for why match-specific rents are not divided efficiently.

Finally, one might think about wage rigidities in a model without any match-specific rents. At a crude level, one could impose a wage above the market-clearing level in an otherwise competitive framework. The aggregate implications for employment, unemployment, and the labor wedge would be very similar to what comes out of a search model. But this approach seems unsatisfactory to us, since the wage rigidity would necessarily have to be ad hoc, and hence potentially not robust to different policy interventions. To develop a deep theory of wage rigidities, one needs a model that sheds light on the forces that prevent wages from adjusting to clear the labor market. The trading frictions inherent in search models seem a promising way of understanding these forces. To the extent that substantial heterogeneity across workers and jobs makes search frictions more pronounced in the labor market than in most other markets, this may help us to understand why rigidities are more important in the labor market as well.

\(^{40}\)For a discussion of these sources of rents, see Manning (2010).
2 Trends

We now shift our attention from cyclical fluctuations to long-run trends. The persistent, widespread, but unequal increase in unemployment across OECD countries in the 1970s and 1980s motivated a substantial body of research that sought to understand why different countries experienced different outcomes. This section reviews some of the key features of the low-frequency data and then examines how search theory has been used to understand these trend changes in labor market outcomes.

2.1 Facts

2.1.1 Unemployment Rate

We start by presenting two key features of trend changes in unemployment across the OECD since 1965.\footnote{Layard, Nickell and Jackman (2005) and Blanchard (2006) are recent contributions that also summarize some facts about unemployment evolutions in the OECD.} First, almost all countries experienced a single peak in trend unemployment during this period. Second, there was substantial heterogeneity in the extent of the increase in trend unemployment and the timing of the peak.
Figure 14 displays the average trend unemployment rate for 17 OECD countries from 1965 until 2007, where the trend is defined as an HP filter with smoothing parameter 100.\footnote{Throughout this section, all OECD averages are unweighted by population. For further details on the data source and countries, see footnote 7.} As the picture shows, trend unemployment increased steadily from the beginning of the sample until the mid 1990s, ultimately rising by a factor of six. This increase was far larger than the changes associated with business cycle fluctuations.

The subsequent decline in trend unemployment was equally important, although even in 2007 the level is more than three times higher than it was in the mid 1960s. Researchers studying unemployment through the mid 1990s took as a starting point that one needed to understand the factors that caused a permanent increase in unemployment. But Figure 14 suggests that the key fact to explain is not a permanent increase in unemployment but rather a long-lived temporary increase.

A particularly striking feature of the data is the heterogeneity in unemployment evolutions across countries. Figure 15 plots the distribution of unemployment rates at five year intervals starting in 1965. An interesting feature of this figure is that as mean unemployment increased from 1965 to 1995, there was a marked increase in the dispersion of unemployment rates as well. The subsequent decrease in unemployment was accompanied by a corresponding decrease in dispersion. In fact, as shown in Figure 16, the standard deviation of unemployment rates in 2007 is roughly the same as it was in 1965 despite the fact that mean unemployment is higher.

Next we look at trend unemployment in each of the individual countries. We note upfront that these figures show two striking feature. First, in almost every country the evolution of trend unemployment followed a single peaked shape, similar to that found for the cross-country average. Second, although the qualitative shape of trend unemployment is the same for all countries, the extent of the increase from initial level to peak level and the timing of the peak varied significantly across countries.

We begin by displaying in Figure 17 the evolution of trend unemployment in France, Germany, the United Kingdom and the United States. This figure illustrates these two general features. While each country followed the same single peaked shape, the magnitude of the increase from initial level to peak level was four percent in the United States but more than nine percent in the other countries. And while the United States reached its peak in the early 1980s and then rose again modestly at the end of the sample, the United Kingdom peaked in the mid 1980s, followed by France in the mid 1990s, and Germany near the end of the sample. Note that there is no strong relationship between the timing of the peak and the
Figure 15: Each dot shows the trend unemployment rate in one OECD country at five year intervals.

Figure 16: The line shows the standard deviation of the trend unemployment rate across OECD countries.
extent of the initial increase. For example, France and the United Kingdom had roughly the same increase up to the peak, but the United Kingdom reaches its peak roughly ten years earlier.

Some of our subsequent analysis will focus on these four economies due to their size and importance in various debates about unemployment. But first it is interesting to note that the features observed for these four countries also hold for every other country in our sample except Portugal. Figures 18–20 show the evolution of trend unemployment for thirteen OECD economies.

To quickly summarize, while in all of the figures we basically see dynamics that follow a single peaked profile, the timing of the peak and the extent of the increase prior to the peak differ quite dramatically across countries. Similarly, the change in unemployment from the beginning to the end of the period was very different across countries. Given that the dynamics of unemployment appear to be persistent but not permanent, it is difficult to say at this point whether these differences will continue to change in the future. For example, in countries such as the United States and Ireland, unemployment is basically the same at the
Figure 18: The four lines show trend unemployment in Spain, Belgium, Italy, and Switzerland.

Figure 19: The four lines show trend unemployment in Finland, Sweden, Denmark, and Norway.
beginning and the end of the period. In contrast, in countries such as France and Germany, unemployment is substantially higher at the end of the period. But given that France and Germany reached their peak unemployment levels much later than the United States, it is unclear whether trend unemployment in these countries will continue to decline.

2.1.2 Total Hours

The large but divergent evolution of the unemployment rate across countries invites an analysis of the sources of these differences. But before we discuss this, we place these trend unemployment changes in a broader context. In our discussion of United States cyclical fluctuations, we concluded that changes in hours per worker account for about a third of the overall volatility in total hours and that the labor force participation rate is comparatively acyclic. This means that changes in unemployment over the business cycle capture a great deal of the change in total hours worked over the business cycle, and so to a first approximation, understanding cyclical fluctuations in total hours amounts to understanding the movement of workers between employment and unemployment. To the extent that search theory informs us how workers move between unemployment and employment, search theory could potentially play a key role in understanding movements in aggregate labor market
outcomes at business cycle frequencies.

We have already established that there are large low frequency changes in unemployment over time, and so now we examine whether these are the dominant source of low frequency changes in aggregate labor market outcomes. We show that low-frequency changes in unemployment account for a relatively small share of the movement in total hours for the period from 1965 to 1995. They have become relatively more important since 1995, because the magnitude of movements in total hours has diminished. Additionally, cross-sectional differences in unemployment currently account for a very small fraction of the overall dispersion in total hours. This suggests that, although search theory has the potential to shed light on the forces that shape the large low frequency movement in unemployment over time, it is not likely to be of first order importance in understanding changes in total hours.

We start by looking at the evolution of the cross-country average of trend total hours for the 17 countries in our sample. Figure 21 shows a dramatic decline in total hours between 1965 and 1995. The magnitude of the drop exceeds 15 percent, which is again much larger than the decline in total hours associated with business cycle fluctuations. There is a striking similarity between the time series data for average unemployment and average total hours. Whereas unemployment increased until the mid 1990s and declined thereafter, total hours decreased until the mid 1990s and increased thereafter. And while unemployment displays a net increase over the entire period, total hours displays a net decrease. Based on a cursory look at the patterns, one might be tempted to conclude that the changes in total hours and changes in unemployment are just two ways of describing the same phenomenon. However, a somewhat closer look reveals some important differences. For example, whereas the decrease in total hours was almost complete as of 1985, the unemployment rate continued to increase sharply after 1985.

A simple decomposition allows a more quantitative assessment of the importance of unemployment rate changes. Our measure of total hours \( H \) is the product of hours per worker \( h \) and the employment-population ratio \( E/P \). And the employment-population ratio can itself be expressed as the product of the participation rate \( PR \) and one minus the unemployment rate \( 1 − UR \). To see this note that if we let the stock of employed, unemployed and total population be denoted by \( E, U, \) and \( P \) respectively, then \( PR = (E + U)/P \) and \( 1 − UR = E/(E + U) \). Total hours can then be expressed as the product of three terms:

\[
H = h \cdot PR \cdot (1 − UR)
\]

The contribution of changes in unemployment to changes in total hours is accounted for by the third term. Figure 22 plots the time series for the trend components of the cross-country
averages for each of these three series, with each value expressed relative to its 1965 value in order to facilitate comparisons. By examining the relative change in each of these three terms over time we can assess the importance of each component.

The figure shows that over the entire period, the increase in the unemployment rate reduced total hours by about four percent. At its peak, in the mid 1990s, the contribution was around seven percent. While this is large relative to changes at business cycle frequencies, it is much smaller than any of the other trend changes that took place in the labor market over this same time period. For example, the increase in labor force participation raised total hours by almost 15 percent, while the decrease in hours per employed worker lowered total hours by 20 percent.

The large trend changes in participation and hours per worker are probably not associated with search frictions. The increase in participation is due entirely to the increased participation of women, and it seems unlikely that search is a key factor in understanding the widespread increase in female participation.\footnote{For example, prominent papers in this literature include Galor and Weil (1996), Goldin and Katz (2002), Jones, Manuelli and McGrattan (2003), Greenwood, Seshadri and Yorukoglu (2005), Olivetti (2006), and Attanasio, Low and Sanchez-Marcos (2008), and none even mentions search as an important element to consider.} And the decrease in hours per worker is
accounted for by increases in vacation days and statutory holidays, decreases in the length of the full-time work week, and increases in part-time work. Once again, search frictions do not seem to be a key element in explaining these trends. We conclude that search is probably not a key element of the explanation for the dramatic decline in hours worked over the entire period since 1965.

To pursue this a bit further, we focus on four individual countries—France, Germany, the United Kingdom and the United States. We begin in Figure 23 by displaying the series for total hours. All three European countries experienced a very significant drop in total hours over this period, ranging between 25 and 35 percent. In contrast, the change in total hours for the United States between 1965 and 2007 was relatively modest.

There are again some qualitative similarities between between the evolution of total hours and unemployment rates for these four economies. The United States had relatively little change in both its unemployment rate and total hours from 1965 to 2007, though in each case there are some low frequency movements between the two endpoints. And for each of the three other countries, there was a net increase in the unemployment rate and a net decrease in total hours over the period.

\footnote{See Pissarides (2007) for an analysis that jointly considers evolutions in unemployment and total hours.}
Once again, these qualitative comparisons at the individual country level might lead one to suspect that changes in total hours were dominated by changes in unemployment. Figures 24–27 decompose low frequency movements in total hours for each of these four countries. While the exact numbers vary a little across countries, these figures confirm the earlier conclusion reached on the basis of cross-country averages—the participation and hours per worker margins were collectively much more important than the unemployment margin in accounting for changes in total hours. In particular, whereas total hours fell between 25 and 35 percent for the three European economies, the decrease accounted for by changes in the unemployment rate was only between four and nine percent.

A related but distinct calculation is to ask how important cross-sectional differences in unemployment are in accounting for cross-sectional differences in total hours. More precisely, we ask what would total hours be in a country relative to the United States if we were to move individuals between employment and unemployment so as to give all countries the same unemployment rate as the United States, but leave hours per worker and labor force participation unchanged. Figure 28 reports the results of such an exercise based on
Figure 24: The three lines show the labor force participation rate $PR$, the employment-labor force ratio $1 - UR$, and hours per worker $h$ in France.

Figure 25: The three lines show the labor force participation rate $PR$, the employment-labor force ratio $1 - UR$, and hours per worker $h$ in Germany.
Figure 26: The three lines show the labor force participation rate $PR$, the employment-labor force ratio $1 - UR$, and hours per worker $h$ in the United Kingdom.

Figure 27: The three lines show the labor force participation rate $PR$, the employment-labor force ratio $1 - UR$, and hours per worker $h$ in the United States.
the 2005 cross-section. Consistent with the earlier calculations, we see that differences in unemployment account for differences in total hours on the order of five percent or less. While differences of this magnitude are quantitatively important from a business cycle perspective, they are relatively small in the context of understanding the cross-sectional dispersion in total hours across countries.

2.1.3 Unemployment Inflows and Outflows

One key feature of search models is that they make predictions about flows into and out of unemployment. In our earlier analysis of business cycle fluctuations, we argued that recessions are characterized by a short, sharp spike in the inflow rate into unemployment and a persistent decline in the outflow rate. We are interested in knowing whether a persistent decline in the unemployment outflow rate also accounted for the substantial increase in the unemployment rate from 1965 to 1995 and the subsequent reversal. Unfortunately, data availability limits the extent to which one can readily carry out such an analysis for a large set of countries over a long time period. Nonetheless, there are data that can shed some light on this issue, and recent work has made some headway in producing estimates for several countries. In this section we summarize this evidence.

The evidence that we present here supports the following three conclusions. First, there
are large differences in unemployment inflow and outflow probabilities across countries that are not related to differences in unemployment rates. Second, in terms of accounting for low frequency changes in unemployment, changes in both inflow and outflow probabilities played a significant role. Third, there does not appear to be a systematic pattern regarding the importance of changes in inflows and outflows that holds across countries. Moreover, for some countries the relative importance of these two flows changes over time.

Our main source of worker flow data is the OECD, which publishes the distribution of unemployment duration for the current stock of unemployed workers. The coverage is incomplete, starting at 1976 in some countries but not until 1983 for many other countries. For the most part, therefore, these results apply to the post 1980 period. We emphasize that many factors can influence unemployment duration distributions, including the prevalence of switches between unemployment and inactivity, the role of temporary layoffs and temporary jobs, the demographic and industrial composition of the workforce, etc. The development of high quality, comparable time series measures of worker flows which controls for variations in these factors is an important issue for future work.

We use data on employment, unemployment, and the fraction of workers who have been unemployed for less than one month to make some inferences about the inflow and outflow probabilities over time for a cross-section of OECD countries. A more thorough analysis can be found in Elsby, Hobijn and Şahin (2008), and we refer the reader to that paper for a detailed discussion of some key issues. To construct the unemployment outflow (or job finding) probability, we use a version of equation (1), but impose steady state, \( u_t = u_{t+1} \) and \( u_t^{<1} = u_{t+1}^{<1} \). Thus the unemployment outflow probability is just \( F_t = u_t^{<1}/u_t \), the fraction of unemployed workers with duration less than one month. Intuitively, in steady state the outflow of new workers balances the inflow, and so the fraction of workers unemployed for less than a month is equal to the unemployment outflow. We use a similar approach to construct the unemployment inflow probability. After correcting for time-aggregation, this gives \( X_t = 1 - (1 - F_t)^{u_t/e_t} \). Elsby, Hobijn and Şahin (2008) show how one can relax the steady state assumptions by using additional information on the stock of unemployed by duration. In the United States case, imposing steady state scarcely affects the behavior of these two time series. More generally, for the points we emphasize here, the steady state assumption is unimportant, and so we present simple estimates that use that assumption.

We focus on data for four countries that illustrate our key points, France, Sweden, the United Kingdom and the United States. Figure 29 presents a scatter plot of the trend unemployment rate against the trend unemployment outflow probability for these four countries. Two features are apparent. First, with the exception of the United States, there is negative

\footnote{For a derivation of this formula, see Shimer (2007).}
Figure 29: Unemployment rate and unemployment outflow probability in France, Sweden, the United Kingdom, and the United States

correlation between trend changes in the unemployment rate and trend changes in the unemployment outflow probability. We will consider this in more detail later on in this section.
Second, even at a given level of trend unemployment, there are dramatic differences in the unemployment outflow probability across countries. For example, all countries in this group experienced a trend unemployment rate of six percent at some point during the sample period. But at this level of trend unemployment, the trend unemployment outflow probability varied from around five percent in France to around forty percent in the United States. Even if we focus on the three European countries, the differences were still enormous, with the probability for the United Kingdom three times as high as the probability in France, and Sweden 1.5 times as large again.

Figure 30 compares the trend unemployment rate with the trend unemployment inflow probability for the same group of countries. Once again, two features are apparent. First, for all countries except the United Kingdom, there is a positive relationship between changes in the trend unemployment rate and changes in the trend unemployment inflow probability. Second, even at a given unemployment rate, there were very large differences in unemployment inflow probabilities. At an unemployment rate of six percent, the unemployment inflow
probability varied by more than a factor of four.

We next consider the issue of how changes in trend unemployment can be decomposed into changes in inflow and outflow probabilities. Elsby, Hobijn and Şahin (2008) argue that for this purpose one should focus on the log of the inflow and outflow probabilities and we follow this practice. Figures 31 and 32 provide scatter plots for unemployment rates and the log of inflow and outflow probabilities for the United States and the United Kingdom. These two figures are of particular interest because they follow very different patterns. In the United States, increases in trend unemployment were associated with effectively no change in the unemployment outflow probability and an increase in the unemployment inflow probability. That is, changes in trend unemployment were entirely accounted for by changes in unemployment inflows. In contrast, the exact opposite is found in the United Kingdom. Here there was effectively no change in the unemployment inflow probability, implying that changes in trend unemployment were accounted for by changes in unemployment outflow probabilities.

Next we consider the dynamics for Sweden, presented in Figure 33. In this case, increases in trend unemployment were accounted for by an increase in the unemployment inflow prob-
Figure 31: Log unemployment inflow and outflow probabilities for the United States.

Figure 32: Log unemployment inflow and outflow probabilities for the United Kingdom.
ability and decrease in the outflow probability that were almost equal in magnitude. Recall that the trend unemployment rate followed a single peaked shape, so that there is an interval of unemployment rates that occurred both during the upswing and during the downswing. It is interesting to note that the relationship between the flows and unemployment appears to have been fairly stable over time within Sweden, in the sense that the flow probabilities associated with a given unemployment rate were independent of whether the unemployment rate was increasing or decreasing.

Finally, we consider France in Figure 34. Similar to the case of Sweden, we see that increases in trend unemployment were accounted for both by a decrease in the outflow probability and an increase in the inflow probability. However, here there was a marked asymmetry between the pattern during the upswing and downswing in (trend) unemployment. Although the points are not labeled by year, the lower portion of the scatter plots for unemployment rates higher than eight percent correspond to the post-1995 period, when trend unemployment was falling. During the pre-1995 period, both factors contributed to the increase in the unemployment rate, although the contribution of the change in the outflow probability was somewhat greater. Indeed, looking more closely, we see that the unemployment inflow probability stopped increasing when unemployment hit approximately eight percent. The subsequent increase in the unemployment rate was accounted for entirely by
a decrease in the outflow probability. After 1995, trend unemployment started to fall. This was entirely accounted for by a decrease in the unemployment inflow probability, with no change in the outflow probability.

In summary, and in contrast to business cycle frequencies, there are no strong empirical regularities on worker flows that hold across all OECD countries. There were large differences in worker flows across countries even when unemployment rates were the same. And in some countries inflow probabilities explained most of the trend movements in unemployment, in other countries outflow probabilities were more important, and in still other countries the importance of the two factors varied over time.

2.1.4 Labor Wedge

In Section 1.1.6, we analyzed the cyclical properties of the labor wedge and argued that, from the perspective of a standard representative household model, recessions are times when it looks like the tax on labor is high. Prescott (2004) uses the same approach to analyze the trend change in hours worked in the G-7 countries between the early 1970s and early 1990s. He verified that variation in hours worked over time and across countries is associated with variation in the labor wedge, but also argued that the wedge was largely accounted for by
measured labor and consumption taxes.

Ohanian, Raffo and Rogerson (2008) extends this analysis to more countries and a longer time period. While they found that changes in labor and consumption taxes account for a large share of the change in the labor wedge for many countries, there are movements in the labor wedge beyond those that can be explained by measured taxes. In some cases the wedge that remained after accounting for labor taxes was positive, suggesting that individuals are not able to work as much as they would like, while in others it was negative, suggesting that individuals were working more than they would like to. Subsequent work has tried to account for these cases as well. Rogerson (2008) and McDaniel (2009) argue that incorporating trend movements from home production to market work (Aguiar and Hurst, 2007; Ramey and Francis, 2009) helps to decrease the absolute size of the labor wedge in countries such as the United States. And Ragan (2004) and Rogerson (2007) argue that modeling detailed features of government spending in Scandinavia, such as child care subsidies, helps to explain the relatively small labor wedge in those countries.

More generally, the fact that changes in labor and consumption taxes account for a large share of the changes in labor wedge is consistent with a model where individuals are on their labor supply curves, after taking taxes into account. This supports our argument that frictions are not a key part of the story behind the large trend changes in hours work.46

2.2 Theory

This section reviews two very distinct ways that search theory has been used to understand the hump-shaped pattern in trend unemployment and the large differences in worker flows. First, some papers have argued that workers’ search decisions play a quantitatively important role in understanding both cross-country differences in worker flows and the different evolutions of aggregate unemployment. For example, if workers choose to look less intensively for jobs and become more selective about which jobs to accept, then holding all else constant, unemployment durations will be longer. Second, other papers have taken advantage of the fact that search creates a tractable framework with match specific rents in order to explore whether wage determination is important in accounting for cross-country unemployment patterns. For example, different labor market policies can influence the manner in which match specific rents are shared and so change the profitability of job creation. This affects unemployment flows and potentially also changes how trend unemployment responds to shocks.

We begin our discussion with a bit of background and context. The initial large and

46Krusell, Mukoyama, Rogerson and Şahin (2009) show that adding empirically plausible search frictions has virtually no impact on the relationship between taxes and total hours.
persistent increase in unemployment in some major European countries generated a nascent literature seeking to explain the phenomenon; see Blanchard (2006) for a recent review. Here we sketch a few details. From the outset, theories included two key components: shocks (or driving forces) and propagation mechanisms. Simply put, something must have changed to increase the unemployment rate, and the increase in unemployment reflected the process through which these changes were propagated through the economy. An important early contribution, summarized in Bruno and Sachs (1985), argued that shocks to oil prices and the slowdown in productivity growth were the key driving forces, and that the failure of real wages to adjust was the key propagation channel. Search theory played very little role in this analysis.

A related research effort, which culminated in the first edition of Layard, Nickell and Jackman (2005), further developed a framework for analyzing how the interaction of shocks and institutions influenced unemployment. Although this analysis did attribute some role to the search behavior of workers, it did not involve any formal modeling of the search and matching process.  

As high unemployment persisted long after the oil shocks had dissipated and beyond the initial decline in productivity growth, researchers looked for other driving forces. Krugman (1994) emphasized broad-based technological change as the shock. The starting point for Krugman’s theory was the observed increase in wage dispersion in the United States, particularly between low and high skill workers. Following many others, Krugman attributed this increased dispersion to skill-biased technological change. He posited that this shock should be present in all advanced economies. In the face of this common shock, labor market responses differed across countries because of differences in labor market institutions. Consistent with the earlier literature, Krugman also emphasized wage setting institutions. But whereas the earlier literature had focused on how wage setting institutions affected the change in the overall wage level, Krugman focused on how institutions affected the change in wage dispersion. In the United States, he argued that wages were largely set in competitive markets, so that skill-biased technical change increased wage dispersion but left unemployment relatively unchanged. In contrast, he argued that in many European economies, wage setting institutions did not allow wages to become more spread out. Instead, unemployment increased for less-skilled workers. Again, this explanation did not attribute any role to search.

Although Krugman’s theory was intuitively appealing, subsequent work by Card, Kra-

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47 Some of this work is summarized in the earlier contributions to this handbook by Johnson and Layard (1986) and Nickell and Layard (1999).
48 See the previous handbook chapter by Katz and Autor (1999) for an excellent survey of this issue.
marz and Lemieux (1999) found little support for the key mechanism in their study of Canada, France and the United States. Still, the dominant approach remains the “shocks-and-institutions hypothesis”: different unemployment evolutions are accounted for by a common shock that is propagated differently across countries because of institutional differences (Blanchard and Wolfers, 2000).\footnote{A notable exception is Daveri and Tabellini (2000), who emphasize cross-country differences both in driving forces and institutions. More recently Nickell, Nunziata and Ochel (2005) have also argued that differences in driving forces are important.} Numerous studies document differences in a variety of labor market institutions, including labor taxes, employment protection, minimum wages, unemployment benefits, and the nature of wage setting. The dramatic differences in unemployment flows across countries even at the same unemployment rate suggests that these institutions affect labor market outcomes; see Nickell and Layard (1999). A different possibility is that unemployment is changing over time across countries because institutions are changing across countries. But Blanchard and Wolfers (2000) found that this was not supported by panel measures of several labor market institutions. In particular, while they argued that there has been some change in institutions over time, these changes account for very little of observed changes in unemployment. On the other hand, they argued that the data does support the view that unemployment changes could be accounted for by common shocks which are propagated differently because of differences in institutions.

While Blanchard and Wolfers (2000) pointed researchers in a particular direction, the paper identified neither the common shock nor the key economic propagation mechanisms. Recent work has sought to isolate the quantitatively important shocks and propagation mechanisms, and in particular the institutional features that impact the propagation of these key shocks. Not surprisingly, since search models have become the dominant models for analyses of unemployment, most of the recent work in this literature has taken place in theoretical models that feature search. Prominent examples include Bertola and Ichino (1995), Ljungqvist and Sargent (1998, 2004, 2007), Marimon and Zilibotti (1999), Mortensen and Pissarides (1999c); Pissarides (2007), and Hornstein, Krusell and Violante (2007). Each of the above papers is also implicitly a theory of cross-country differences in unemployment flows. Several other papers have used search theory to explore unemployment flows without necessarily addressing the issue of how unemployment has evolved over time; see Bertola and Rogerson (1997), Garibaldi (1998), Blanchard and Portugal (2001), Kugler and Saint-Paul (2004), and Pries and Rogerson (2005).

It is beyond the scope of this chapter to review this large literature. Instead, we focus on two examples which illustrate the two roles that search theory has played. Ljungqvist and Sargent (1998, hereafter referred to as LS) focus on the role of worker choices regarding...
search. Hornstein, Krusell and Violante (2007, hereafter HKV) focus on how wage determination affects the profitability of job creation. A closely related point, also of interest in a broader sense, is that the two papers propose fundamentally different views of the underlying economic forces that have led to higher unemployment. While LS focus on the role of worker choices, with firms playing a passive role, HKV focus on the choices of firms, with workers playing a passive role.

2.2.1 Ljungqvist and Sargent (1998)

LS quantify a reasonable version of the shocks-and-institutions hypothesis. Their common shock is an increase in “turbulence.” In their language, when a worker is laid off from her job in a turbulent economy, her human capital often becomes less valuable or depreciates. A simple example serves to illustrate what they have in mind. In one scenario, a worker in an auto assembly plant loses her job due to lower demand for the particular model produced by that plant, but then finds employment at a different auto assembly plant that is hiring workers because of increased demand for its particular model. Although the worker may suffer a spell of unemployment, she suffers no long term wage loss. In a second scenario, there is a permanent decrease in employment in all domestic auto assembly plants, perhaps because of labor-saving technological change or competition from lower-cost assemblers in other countries. In this case, the laid-off worker not only loses her job and experiences a spell of unemployment, but also suffers a drop in expected future wages since the demand for her skills has fallen. LS argue that this second scenario became more prevalent in all OECD countries starting some time in the late 1970s and early 1980s.

LS interact this common shock with differences across countries in social insurance schemes for dealing with workers who suffer job losses. Although they label the policies “unemployment insurance,” they interpret them broadly to include other programs for displaced workers, such as disability and early retirement. In their analysis, LS focus on two different institutional regimes. In one, which they label laissez-faire, there is no social insurance at all. In the other, which they label the welfare state, a worker who loses her job can collect a transfer payment equal to 70 percent of her pre-layoff wage as long as she remains jobless, subject only to the proviso that she cannot reject a job that offers at least as high as a wage as she earned before the layoff.

Frictionless Model  Our goal is to assess whether search frictions per se improve our understanding of low frequency changes in labor market outcomes, and so we compare frictional

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50One issue in terms of connecting this analysis with the data is that individuals in these programs will typically not be classified as unemployed.
and frictionless version of the LS model. We start with the simpler frictionless model. There is a unit mass of workers, each of whom seeks to maximize a utility function of the form:

$$\sum_{t=0}^{\infty} (\beta(1 - \alpha))^t y_t,$$

where $\beta$ is the discount factor, $\alpha$ is the probability of death (assumed constant), and $y_t$ is after-tax income in period $t$. When an individual dies, she is replaced by a newborn individual. Note that individuals do not value leisure. This means that, even though the analysis emphasizes the importance of labor supply responses, these responses are not based on substitution between leisure and consumption.

The key feature of the LS model is learning-by-doing when working and skill depreciation when not working. Let $h$ denote the human capital or skill level of a worker. A newborn worker starts with some minimal level of human capital, which we normalize to 1. Subsequently, there are three transition functions that describe how human capital evolves. Let $\mu_e(h, h')$ be the probability that a worker who remains employed goes from human capital $h$ to human capital $h'$. Similarly, let $\mu_n(h, h')$ be the probability that a worker who is currently not employed goes from human capital $h$ to human capital $h'$. Finally, to capture turbulence as defined above, let $\mu_l(h, h')$ be the probability that an employed worker who gets laid off goes from human capital $h$ to human capital $h'$. In the economy without turbulence, there is no loss in human capital upon layoff.

LS assume a finite set of levels of human capital and parameterize the stochastic human capital accumulation process as follows: when employed, human capital either stays the same or improves by one level. When not employed, human capital either stays the same or deteriorates by one level. It follows that human capital for a continuously employed (non-employed) individual is weakly increasing (decreasing). Finally, when a worker with human capital $h$ is laid off in the turbulent economy, there is some probability that her human capital remains unchanged, but with the remaining probability the worker receives a draw from a distribution with support on $[1, h]$.

There is an unlimited supply of identical jobs, each of which offers a wage $w$ per unit of human capital. That is, a worker with human capital $h$ earns $wh$ if she works. We introduce the base wage $w$ to ease the comparison with the search theoretic version of LS, but it plays no role in the frictionless model. When a worker is laid off, she can immediately move to a new job if she wants. This means that, while layoffs can have a negative consequence for human capital, they do not affect the opportunity to work.

The final feature of the economic environment is a balanced-budget unemployment insurance system, financed by a proportional tax on income. The laissez-faire economy has no
unemployment insurance, while the welfare state economy pays a laid-off worker 70 percent of her pre-layoff earnings until the worker opts to return to work.

LS study four steady state outcomes: the laissez-faire and welfare state, each with and without turbulence. First consider the laissez-faire economy without turbulence, so human capital is unchanged following a layoff. In equilibrium, all individuals work until they die. This follows immediately from the fact that individuals do not value leisure and always have access to a job that offers positive earnings. The equilibrium of the welfare state without turbulence is the same. Although in this economy a laid-off individual has access to life-long unemployment benefits equal to 70 percent of her pre-layoff earnings, she can always find a job that offers 100 percent. Since she does not value leisure, it is optimal to work. The basic message is that, in some environments, large differences in unemployment insurance do not affect equilibrium employment.

Next we repeat this analysis when there is turbulence in the economy. The equilibrium employment decisions in the laissez-faire economy are unchanged. Even though a laid off worker may experience a large negative shock to her human capital, and thereby face lower earnings prospects, it is still optimal for her to work, since leisure has no value. But the equilibrium outcome in the welfare state economy is potentially affected. If a worker experiences more than a 30 percent reduction in her human capital, then the one period return to working is now lower than the one period return to collecting unemployment insurance. Of course, working allows the individual to accumulate human capital, and so it may still be optimal for her to work. But in general, a laid-off worker will stop working if the shock is sufficiently bad. If she does not work in the period following the layoff, then her human capital starts to depreciate. Since unemployment benefits last forever, she will never find it optimal to return to work.

This discussion highlights two key points. First, the impact of unemployment insurance, and other social insurance programs more generally, on aggregate employment depends on the underlying economic environment. In the absence of turbulence, even very generous programs need not affect aggregate employment. Second, the LS model offers an example in which a common shock to economies that have different labor market institutions can be propagated very differently, with very different effects on aggregate employment. This holds even in the absence of frictions.

Model With Search Frictions We now extend the model to allow for search frictions in the spirit of Lippman and McCall (1976), as in the original LS article. A worker without a job makes a decision regarding search intensity ($s$) that influences the probability of receiving a wage offer $w$ draw from the cumulative distribution function $F$ with bounded support. The
probability that a worker receives an offer is an increasing function $\pi(s)$, while the utility cost of searching while unemployed is an increasing function $c(s)$. There is no on-the-job search. Note that a worker’s income is the product of her wage and human capital, $wh$. The wage stays constant on the job, but her human capital may increase. In the welfare state, unemployment benefits are tied to past income.

Search frictions affect model outcomes both through search intensity and through the reservation wage. To understand how, note that in a standard worker search problem without human capital, unemployment insurance reduces search intensity and raises the reservation wage, both of which lead to longer unemployment durations and hence higher unemployment. These forces are present even in this more complicated environment.

This observation has two implications. First, it implies that even in the absence of turbulence, the welfare state economy has higher unemployment than the laissez-faire economy and that this higher unemployment is driven by differences in duration, although the effects are quantitatively modest.

Second, the behavioral response through search intensity and reservation wages amplifies the effect of turbulence. Recall that in the frictionless model, a worker who experiences a sufficiently large shock when laid off becomes non-employed forever. With search frictions and skill depreciation while nonemployed, human capital continues to fall after the initial layoff. This means that, even if the initial shock did not leave the worker preferring non-employment to working, she may slip into this absorbing state during a long unemployment spell. Indeed, if she approaches a point at which she prefers unemployment to work at any wage, her search intensity falls and reservation wage rises, so the job finding probability falls smoothly to zero.

The discussion so far has been entirely qualitative. A key contribution of LS is to assess the quantitative importance of these factors. To do this, LS compute the steady state equilibria for each of the two UI regimes both with and without turbulence. The no-turbulence laissez-faire economy is parameterized so as to match several features of the United States economy in the 1970s. LS assume that all economies share the same preferences and technologies, and that the only difference is the unemployment insurance system.

In the absence of turbulence LS find that both economies have similar steady state unemployment rates, although long duration unemployment is more prevalent in the welfare state economy with the UI system. This comparison is important, since as we noted previously, unemployment duration was higher in France and Sweden than in the United States, even when unemployment rates were similar.\footnote{In fact, unemployment was actually lower in Europe than in the United States in 1970. Ljungqvist and Sargent (2008) show how to modify their analysis to capture this feature.} But when they introduce turbulence, they find
that the welfare state moves to a steady state with much higher unemployment, whereas unemployment in the laissez-faire economy changes relatively little. In addition, all of the increase in unemployment is accounted for by increased duration of unemployment. Based on these findings, LS conclude that an increase in turbulence combined with different unemployment insurance systems is quantitatively important in accounting for the variation in unemployment rate evolutions across countries.

As we noted before, the higher unemployment that appears in the welfare state in turbulent times is purely the result of worker choices. Indeed, there are no firms in the model. In later work, Ljungqvist and Sargent (2004) extend the framework to an environment with endogenous job creation and show that their results are basically unchanged. For our purposes, the most important observation is that one can in principal obtain similar results even without search frictions.

2.2.2 Hornstein, Krusell and Violante (2007)

HKV propose an alternative version of the shocks-and-institutions hypothesis. In this case, the shock is an acceleration in capital embodied technological change, consistent with the acceleration of the decline in equipment prices from the 1960s to the 1990s documented by Gordon (1990) and Cummins and Violante (2002).\footnote{Earlier work on technological change in the context of search models includes Aghion and Howitt (1994) and Mortensen and Pissarides (1999c).} As in LS, HKV examine the effect of different unemployment benefit systems, but also broaden the set of relevant institutions to include labor income and firing taxes. They find that the effect of an increase in the pace of technological change depends on how wages behave, which in turns depends on the amount of rents in the initial equilibrium, itself a function of these institutions.

Frictionless Model  We again start with a version of the model that does not include search so as to better understand what role search plays in the HKV’s analysis. There is a unit mass of identical workers, each with preferences

\[
\int_0^\infty e^{-\rho t} (c(t) - \gamma n(t)) dt,
\]

where \(c(t)\) is consumption at time \(t\), \(n(t) \in \{0, 1\}\) is labor supply at time \(t\), and \(\rho\) is the discount rate. The unit of production is a matched worker-machine pair, so the microeconomic production technology is Leontief in machines and labor. Machines are indexed by age and a matched worker-machine pair with age of machine \(a\) produces output \(e^{-ga}\). Output of a new machine is normalized to one. The parameter \(g\) embodies two forces: depreciation and
capital embodied technological change. Relative to the newest vintage, both of these forces imply a negative effect of age on productivity. A new machine costs $I$ units of output. There is an unbounded set of potential entrants. A key assumption is that machines cannot be upgraded. The scrap value of a machine is normalized to zero.

Consider the steady state competitive equilibrium in this economy. There are two markets, one for output and one for labor. Let output be the numeraire and let $w$ denote the wage. Because this model features linear technology and preferences, equilibrium will generically entail either everyone working or no one working. The only interesting case is the one in which everyone works, so in what follows we will assume that parameters are such that this is true in equilibrium.

The only real decision in this economy is the age at which a machine gets scrapped. Given a wage of $w$, a machine is scrapped at the age $a$ solving $w = e^{-ga}$, since this is the point at which it is no longer profitable to operate the machine. Given that there is an unbounded set of potential entrants, the equilibrium $w$ is the one at which the present discounted value of profit from entering is zero. This implies

\[ \int_0^{\frac{\log w}{g}} e^{-ra} (e^{-ra} - w) da = I, \]

so the present value of profits during the productive life of the machine just covers the initial cost of the machine. This equation uniquely determines $w$.

Now consider what happens to the steady state equilibrium of this economy if there is an increase in the rate of obsolescence, i.e., an increase in $g$. Implicitly differentiating the above expression shows that $w$ must fall. The basic intuition is that if a machine becomes obsolete more rapidly then it will be used for a shorter period of time, implying that wages must decrease in order to maintain the profitability of investment. It also follows that the scrapping age will decrease. The same must be true if there is an increase in the rate of technological change: the wage relative to the productivity of the frontier technology must fall, and the scrapping age decreases as well. In the economy without search frictions, this has no consequences for unemployment or employment. Workers move directly from a machine that is being scrapped to a new machine. There will, however, be more reallocation of workers when $g$ is larger. But the key point is that in order to maintain equilibrium in the labor market, increases in the pace of technological change necessitate a decrease in wages relative to the productivity of the frontier technology.

\[ ^{53}\text{We follow HKV in measuring everything relative to the frontier technology so as to make the economy stationary. This means that in our exposition, the wage is constant relative to the frontier, and so is actually growing over time if there is capital embodied technological change.} \]
Model With Search Frictions  Now we return to the full HKV model with search frictions. Similar to the model in Section 1.2, frictions are modeled through the use of a constant returns to scale matching function, but here the inputs are unmatched workers and machines. There are no search costs, only the upfront cost of a new machine. Workers separate from matches for exogenous reasons at rate $x$. The machine still exists after a separation of this sort and will look for a new worker.

Wage determination is the key to HKV’s analysis. The presence of search frictions gives rise to match specific rents and there are many ways that these can be divided. Following the approach pioneered by Pissarides (1985) and Mortensen and Pissarides (1994), HKV assume that wages satisfy the Nash bargaining solution. This gives rise to a wage function $w(a)$. In particular, and in contrast to the frictionless model, workers who work with machines of different vintages are paid different amounts, since the rents associated with a given match depend on the age of the machine.

As before, free entry ensures that in equilibrium firms earn zero expected profits from the purchase of a new machine. But two endogenous factors now influence profits: the wage function and the fraction of time that the machine will be idle. Increased idleness and higher wages both reduce profits. Zero profit can be achieved either with low idleness and high wages or high idleness and low wages, or combinations in between. Note that, with a constant returns to scale matching function $m(u, v)$, a decrease in the idleness of machines can occur only if unemployed workers find vacant machines at a lower rate, thereby increasing unemployment duration. Thus equilibrium requires either higher unemployment and higher wages or lower unemployment and lower wages. What happens depends on the wage function $w(a)$.

With Nash bargaining, the response of wages to an increase in the rate of skill-biased technical change $g$ is influenced by various labor market policies. Intuitively, wages can be thought of as having two components, one associated with the worker’s outside option and the other with the worker’s share of the match specific rents. As the match specific rents become smaller, rent-sharing represents a smaller fraction of the total wage. As a result, the total wage responds less to $g$ and frictions respond more. Labor income taxes and unemployment benefits tend to reduce the surplus. To the extent that these policies are more prevalent in Europe than the United States, there is less wage adjustment in Europe and so more adjustment in the amount of time that workers spend idle.

Once again, a key issue is the quantitative importance of this mechanism. HKV calibrate the model to match features of the United States economy in the early 1970s. They then solve for the steady state that corresponds to policy settings that represent a “typical” European economy, assuming that Europe is identical to the United States in all non-policy factors
except $x$, the rate at which workers separate from jobs for exogenous reasons. They then increase the pace of technological change to the rate observed in the 1990s and solve for the steady state equilibrium for both the United States and European economies. Their key finding is that unemployment is much more responsive to the pace of technological change in Europe than in the United States. The outflow from unemployment falls, due to the decrease in investment in new machines, and the inflow rate increases, due to the reduction in the scrapping age. Quantitatively, HKV find that the reduction in outflows accounts for most of the increase in unemployment.

As noted earlier, and in contrast to the analysis in LS, increased unemployment in this model is attributed entirely to firms’ actions. Workers are completely passive, accepting any job that they are offered. Unemployment increases primarily because firms create fewer new machines and so post fewer vacancies. But the main role played by search is that it gives rise to match-specific rents. Differences in how these rents are divided are important for the behavior of aggregate unemployment.

### 2.2.3 Discussion

The main goal of this section has been to highlight how search theory has been used to understand cross-country differences in labor market outcomes. Although we discussed two papers in more detail, we view these as representative of a larger literature that has sought to flesh out the shocks-and-institutions hypothesis. In closing, we discuss how well these papers account for the key features of the data that we have previously described. We focus on four dimensions.

First, both models look only at the movement of workers between employment and unemployment; neither has a participation margin or an hours margin. We previously documented that the unemployment margin accounts for a relatively small fraction of the overall changes in total hours, which raises the issue of whether it is appropriate to start with a framework that abstracts from the quantitatively more important margins. More generally, given that something must be generating these large changes in hours and participation, it is unclear if it is reasonable to ignore these changes when thinking about unemployment.

Second, both models solve for two steady state equilibria, one that corresponds to the “earlier” period and the other that corresponds to the “later” period. But as we documented previously, it does not look like Europe moved from a low unemployment steady state to a high unemployment steady state. Instead, it seems more promising to think of a shock that

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54 As noted above in the discussion of the LS model, a key fact is that unemployment in Europe was not always higher than in the United States, despite the fact that European institutions have remained relatively constant. The lower value of $x$ in Europe is critical in allowing for the HKV model to be consistent with this fact.
led to a long-lived but at least partially temporary increase in unemployment. One possibility is that the shocks themselves are long-lived but temporary, but at a minimum this points to the need for better measurement of the key forcing processes. For example, LS provide indirect evidence on the increase in turbulence, based on work by Gottschalk and Moffitt (1994) that decomposes earnings in the 1970s and 1980s into transitory and permanent components. LS show that their calibrated model can replicate the key features of the change in the decomposition in the United States. While this decomposition provides some useful information, Gottschalk and Moffitt (1994) do not directly measure the wage losses associated with layoffs. Nor has anyone checked whether the changes in the decomposition were reversed after 1995, when unemployment started to fall throughout the OECD. Finally, both LS and HKV impose that the shocks are the same across countries, without offering direct evidence on this point. That is, they do not directly test the shocks-and-institutions hypothesis.

Third, both models contrast outcomes in two economies, the United States and a “typical” European economy. But a key feature of our earlier data analysis is that there is a large amount of heterogeneity in unemployment dynamics even across European economies, both in terms of the magnitude of the increase and in terms of the timing of the peak. It is also well known that there is substantial heterogeneity in institutions and policies across economies. Accounting for the heterogeneity in outcomes across countries is an important test for any theory of unemployment dynamics.

Fourth, both papers focus entirely on aggregate outcomes. There is potentially a lot of useful information in the disaggregated data that may help to distinguish competing theories. Although we have not presented any detailed information about the patterns of labor market outcomes in disaggregated data, we note two findings here: unemployment differences are particularly pronounced among younger workers, while employment differences are particularly pronounced both for younger and older (but not prime-aged) workers. More generally, incorporating differences in age, gender, and skill level may be useful for distinguishing theories of labor market outcomes.55

Finally, we want to address the role of search in these two papers. While LS emphasize workers’ choices about search intensity and reservation wages, they do not show that search frictions are important for propagating turbulence. To our knowledge, this question remains unanswered. More generally, whether search intensity amplifies or mutes the response of the economy to shocks may depend on the nature of the shock. For example, the unemployment consequences of shocks that require reallocating workers across sectors may be muted by the possibility of searching more intensively.

55Kitao, Ljungqvist and Sargent (2009) is a recent example that moves in this direction.
The analysis of HKV raises several interesting questions. First, we have emphasized that differences in rent-sharing is a key mechanism in their model. While search models represent a tractable framework that gives rise to match specific rents, we previously noted that other mechanisms may also give rise to match-specific rents. We know of no attempt to measure whether search frictions are an important source of match-specific rents, nor of any work that has examined whether the source of rents is important for labor market outcomes.

Second, note that HKV shares a common feature with Bruno and Sachs (1985) and Krugman (1994), in that all three papers emphasize that unemployment increases when wage setting institutions prevent wages from responding sufficiently following a particular shock. A key difference is that HKV assume that wage setting institutions are fundamentally the same across economies and over time, represented by a Nash bargaining solution with the same bargaining weights in each country and at each point in time. Wage outcomes differ because of the interaction between the wage setting institution and other labor market policies. Two interesting and open questions are the extent to which different economies in fact have different wage setting processes and, if they do, how to model those differences. This is of course not inconsistent with search theory. Indeed, one nice feature of the Mortensen and Pissarides (1994) framework is that it can easily accommodate a wide variety of specifications for wage setting. Nonetheless, little work has sought to assess the extent to which different countries should be modeled as having different wage setting processes. Recent work that leads in this direction includes Mortensen (2003), who asks which of two wage setting specifications better fits Danish microeconomic data, and Hall and Krueger (2008), who use survey evidence to assess the relevance of different wage setting procedures in the United States economy.

3 Conclusion

Our objective in this chapter has been to explore how the explicit introduction of search frictions into otherwise standard macroeconomic models affects our understanding of aggregate labor market outcomes in two different contexts. In our analysis of business cycles, we found that the search framework is useful for interpreting facts about unemployment and labor market flows. But we also found that search frictions tend to dampen fluctuations in output and employment without significantly increasing their persistence. Moreover, by dampening employment fluctuations, search frictions cause a counterfactual procyclical labor wedge. In addition, using search theory did not lead us to introduce any important new

[56]See the handbook chapter by Mortensen and Pissarides (1999c) for details on how different models of wage determination can be embedded in search-theoretic models of the labor market.
shocks into the neoclassical growth model. On all of these counts, search behavior itself
does not seem intrinsically important for business cycle analysis. On the other hand, search
provides a promising environment for studying the implications of alternative wage setting
mechanisms, which many authors have argued are important at business cycle frequencies.
We remain hopeful that search will be an important component in understanding business
cycle fluctuations. In particular, we think that the ability of search models to connect with
data on job and worker flows will help to discriminate between alternative theories of wage
setting behavior.

We also found some striking low-frequency patterns in unemployment and labor market
flows. For example, trend unemployment initially rose across the OECD but has since
fallen in almost every country. And there is substantial heterogeneity in the relationship
between unemployment and worker flows across countries and over time. Search is useful
for interpreting these facts. But unlike in the business cycle analysis, changes in trend
unemployment are typically much smaller than changes in hours per worker and labor force
participation. To the extent that search models lead one to focus only on unemployment,
we feel that the emphasis is misplaced. Still, many recent explorations of the shocks-and-
institutions hypothesis have taken place in models that feature search. While the role of
search frictions per se in these models is unclear, we are also hopeful that these models will
prove useful for understanding the extent to which differences in wage setting institutions as
well as other labor market institutions and policies are an important cause of differences in
labor market outcomes across countries.

In interpreting our conclusions it is important to emphasize that we have focused through-
out on the role of search in macroeconomic models of the labor market. But we would be
remiss not to mention at least briefly the important role that search frictions play in four
distinct branches of the microeconomic literature on labor market outcomes.57

First, search theory has served as the foundation for the analysis of optimal unemploy-
ment insurance. Shavell and Weiss (1979) explored the optimal provision of unemployment
insurance to a worker who must both choose how intensively to search and what wages to
accept. Most of the subsequent literature has focused on the search intensity margin (e.g.,
Hopenhayn and Nicolini, 1997; Chetty, 2008), although a few papers have looked at how
unemployment insurance affects reservation wages (Hansen and İmrohoroğlu, 1992; Shimer
and Werning, 2008). Search is fundamental to these papers, since each needs a model with
unemployment and a moral hazard problem in an environment with idiosyncratic risk. More
generally, models that feature search have been used to analyze many labor market policies.

57Search frictions have also been used to model monetary exchange, housing, marriage, and over-the-
counter asset markets, among other topics.
Ljungqvist (2002), for example, shows that a model with search frictions has distinctive implications for the effects of employment protection on aggregate employment.

Second, search models have been useful in accounting for worker flows. There is a long tradition of using single agent search theory to account for individual level data on unemployment spells and wages. Search behavior plays an essential role in this literature. More recently, equilibrium search models have been used to study data on turnover and wage dynamics. Many are based on the Burdett and Mortensen (1998) model of on-the-job search; prominent examples include Postel-Vinay and Robin (2002), Mortensen (2005), and Cahuc, Postel-Vinay and Robin (2006). These papers take advantage of the availability of large administrative matched worker-firm panel data sets to estimate the structural parameters of the model and use the data to test alternative theories of wage determination. See Lentz and Mortensen (2010) for a recent survey of this literature.

Third, a number of authors have argued that search frictions may play an important role in understanding the evolution of wage inequality. For example, Acemoglu (1999) presents a model where an increase in the supply of skilled workers may change the composition of jobs from a “pooling” equilibrium, in which firms facing search frictions create jobs suitable for all workers, to a “separating” equilibrium, in which firms create different types of jobs for different workers, leading to an increase in between-group wage inequality. Other authors have explored the extent to which search models, by permitting violations of the law-of-one-price, can generate within-group wage inequality. Hornstein, Krusell and Violante (2009) argue that within a broad class of search models, the possibility of an economically significant amount of within-group inequality is limited by the possibility of waiting for better job opportunities. On the other hand, Marimon and Zilibotti (1999) show how the interaction between search frictions and unemployment benefits can generate substantial inequality across ex ante identical workers. Shimer and Smith (2000), Shi (2002), and Shimer (2005a) develop theoretical search models with heterogeneous workers and firms that make predictions for both within- and between-group inequality. These models have been explored empirically, for example by Abowd, Kramarz, Lengermann and Pérez-Duarte (2004) and Lopes de Melo (2009).

Finally, a few papers have used search frictions to address existing issues in the labor contracting literature. In particular, building on Gale (1996) and Inderst and Wambach (2001, 2002), Guerrieri, Shimer and Wright (2009) show that search can naturally resolve problems related to the existence and uniqueness of equilibrium in adverse selection models. Their applications include a labor market rat race, where high productivity workers agree

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58Early examples include Flinn and Heckman (1982) and Wolpin (1987). See also the discussions in Mortensen (1987) and Eckstein and Van den Berg (2007) and the references contained therein.
to an inefficiently high level of hours and employment in order to separate themselves from less productive workers, and the absence of private insurance against the risk of a layoff, where high productivity workers are willing to work without insurance in order to indicate that they are not concerned with this eventuality. More generally, one might expect search frictions to interact with standard issues that arise in labor contracting.

In concluding, it is useful to return to our discussion in the introduction regarding the three different ways that search might matter in macroeconomic models. At this point, we do not see much evidence that search behavior per se is of first order importance in understanding aggregate outcomes in either of the contexts that we considered. However, adding search to otherwise standard macroeconomic models definitely expands the ability of these models to connect with various pieces of empirical evidence. And models that feature search do create a useful framework in which to consider various wage setting mechanisms. Further clarifying the role of search in assessing substantive issues involving the aggregate labor market is an important task for future research.
References


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