Problems

1. The amount of land increases, and, at first, the size of the population is unchanged. Therefore, consumption per capita increases. However, the increase in consumption per capita increases the population growth rate, see the figure below. In the steady state, neither $c^*$ nor $I^*$ are affected by the initial increase in land. This fact can be discerned by noting that there will be no changes in either of the panels of Figure 6.8 in the textbook.
2. A reduction in the death rate increases the number of survivors from the current period who will still be living in the future. Therefore, such a technological change in public health shifts the function $g(c)$ upward. In problem #1 there were no effects on the levels of land per capita and consumption per capita. In this case, the $g(c)$ function in the bottom figure below shifts upward. Equilibrium consumption per capita decreases. From the top figure below, we also see that the decrease in consumption per capita requires a reduction in the equilibrium level of land per capita. The size of the population has increased, but the amount of available land is unchanged.
3. For the marginal product of capital to increase at every level of capital, the shift in the production function is equivalent to an increase in total factor productivity.
   (a) The original and new production functions are depicted in the figures below.

   ![Production Function Graph](image)

   (b) Equilibrium in the Solow model is at the intersection of \( szf(k) \) with the line segment \((n + d)k\).
   The old and new equilibria are depicted in the bottom panel of the figure above. The new equilibrium is at a higher level of capital per capita and a higher level of output per capita.

   (c) For a given savings rate, more effective capital implies more savings, and in the steady state there is more capital and more output. However, if the increase in the marginal product of capital were local, in the neighborhood of the original equilibrium, there would be no equilibrium effects. A twisting of the production function around its initial point does not alter the intersection point.

4. An increase in the depreciation rate acts in much the same way as an increase in the population growth rate. More of current savings is required just to keep the amount of capital per capita constant. In equilibrium output per capita and capital per capita decrease.
5. A destruction of capital.
   (a) The long-run equilibrium is not changed by an alteration of the initial conditions. If the economy started in a steady state, the economy will return to the same steady state. If the economy were initially below the steady state, the approach to the steady state will be delayed by the loss of capital.
   (b) Initially, the growth rate of the capital stock will exceed the growth rate of the labor force. The faster growth rate in capital continues until the steady state is reached.
   (c) The rapid growth rates are consistent with the Solow model’s predictions about the likely adjustment to a loss of capital.

6. A reduction in total factor productivity reduces the marginal product of capital. The golden rule level of capital per capita equates the marginal product of capital with \( n + d \). Therefore, for given \( n + d \), the golden rule amount of capital per capita must decrease as in the figure below. Therefore the golden rule savings rate must decrease.

7. Government spending in the Solow model.
   (a) By assumption, we know that \( T = G \), and so we may write:
   \[
   K' = s(Y - G) + (1 - d)K = sY - gN + (1 - d)K
   \]
   Now divide by \( N \) and rearrange as:
   \[
   k'(1 + n) = szf(k) - sg + (1 - d)k
   \]
   Divide by \( (1 + n) \) to obtain:
   \[
   k' = \frac{szf(k)}{(1 + n)} - \frac{sg}{(1 + n)} + \frac{(1 - d)k}{(1 + n)}
   \]
Setting $k = k'$, we find that:

$$szf(k') = sg + (n + d)k'.$$

This equilibrium condition is depicted in the figure below.

(b) The two steady states are also depicted in the figure above.
(c) The effects of an increase in $g$ are depicted in the bottom panel of the figure above. Capital per capita declines in the steady state. Steady-state growth rates of aggregate output, aggregate consumption, and investment are all unchanged. The reduction in capital per capita is accomplished through a temporary reduction in the growth rate of capital.

8. The golden rule quantity of capital per capita, $k^*$, is such that $MP_k = zf'(k^*) = n + d$. A decrease in the population growth rate, $n$, requires a decrease in the marginal product of capital. Therefore, the golden rule quantity of capital per capita must increase. The golden rule savings rate may either increase or decrease.
9. (a) First, we need to determine how $bN$ evolves over time:

$$ \frac{d(bN)}{} = (1 + f)(1 + n) \ bN $$

Then we just need to redo the analysis of the competitive equilibrium and the steady state as in the book, replacing every $N$ by $bN$, every $(1 + n)$ by $(1 + f)(1 + n)$, and every $n$ by $f + n$. The new steady-state per efficiency unit capital is then

$$ k^* = \frac{szf(k^*)}{(1 + f)(1 + n)} + \frac{(1 - d)k^*}{(1 + f)(1 + n)} $$

All aggregate variables then grow at the rate of $f + n$, while per capita aggregates grow at the rate $f$.

(b) An increase in $f$ increases the growth rate of per capita income by the same amount, as $f$ is its growth rate. This happens because the exogenous growth in $b$ raises instant capital and income for everyone without a need to invest in capital.

10. Production linear in capital: $Y = zK = zf(k) \Rightarrow f(k) = k$

(a) Recall Equation (20) from the text, and replace $f(k)$ with $k$ to obtain:

$$ k' = \frac{(sz + (1 - d))}{(1 + n)} k $$

Also recall that $\frac{Y}{N} = zk \Rightarrow k = \frac{1}{z} \frac{Y}{N}$ and $k' = \frac{1}{z} \frac{Y'}{N'}$. Therefore:

$$ \frac{Y'}{N'} = \frac{(sz + (1 - d))}{(1 + n)} \frac{Y}{N} $$

As long as $\frac{(sz + (1 - d))}{(1 + n)} > 1$, per capita income grows indefinitely.

(b) The growth rate of income per capita is therefore:

$$ g = \frac{\frac{Y'}{N'} - \frac{Y}{N}}{\frac{Y}{N}} = \frac{(sz + (1 - d))}{(1 + n)} - 1 $$

$$ = \frac{sz - (n + d)}{(1 + n)} $$

Obviously, $g$ is increasing in $s$.

(c) This model allows for the possibility of an ever-increasing amount of capital per capita. In the Solow model, the fact that the marginal product of capital is declining in capital is the key impediment to continual increases in the amount of capital per capita.
11. Solow residual calculations.

(a) To calculate the Solow residuals, we apply the formula, 
\[ \dot{z} = \dot{Y} - 0.36 \dot{K} - 0.64 \dot{N} \], 

to the values in the provided table. Adding a new column for these values, we obtain:

<table>
<thead>
<tr>
<th>Year</th>
<th>( \dot{Y} )</th>
<th>( \dot{K} )</th>
<th>( \dot{N} )</th>
<th>( \dot{z} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1995</td>
<td>8031.7</td>
<td>25487.3</td>
<td>124.9</td>
<td>9.478</td>
</tr>
<tr>
<td>1996</td>
<td>8328.9</td>
<td>26222.3</td>
<td>126.7</td>
<td>9.640</td>
</tr>
<tr>
<td>1997</td>
<td>8703.5</td>
<td>27018.1</td>
<td>129.6</td>
<td>9.823</td>
</tr>
<tr>
<td>1998</td>
<td>9066.9</td>
<td>27915.9</td>
<td>131.5</td>
<td>10.019</td>
</tr>
<tr>
<td>1999</td>
<td>9470.3</td>
<td>28899.9</td>
<td>133.5</td>
<td>10.236</td>
</tr>
<tr>
<td>2000</td>
<td>9817.0</td>
<td>29917.1</td>
<td>136.9</td>
<td>10.312</td>
</tr>
<tr>
<td>2001</td>
<td>9890.7</td>
<td>30793.4</td>
<td>136.9</td>
<td>10.282</td>
</tr>
<tr>
<td>2002</td>
<td>10048.8</td>
<td>31599.6</td>
<td>136.5</td>
<td>10.369</td>
</tr>
<tr>
<td>2003</td>
<td>10301.0</td>
<td>32426.2</td>
<td>137.7</td>
<td>10.472</td>
</tr>
<tr>
<td>2004</td>
<td>10703.5</td>
<td>33304.9</td>
<td>139.2</td>
<td>10.703</td>
</tr>
<tr>
<td>2005</td>
<td>11048.6</td>
<td>34191.7</td>
<td>141.7</td>
<td>10.820</td>
</tr>
</tbody>
</table>

(b) Next, we compute the percentage changes in each of the table entries. These values are presented in the table below.

<table>
<thead>
<tr>
<th>Year</th>
<th>( \Delta \dot{Y} ) (%)</th>
<th>( \Delta \dot{K} ) (%)</th>
<th>( \Delta \dot{N} ) (%)</th>
<th>( \Delta \dot{z} ) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1996</td>
<td>3.70</td>
<td>2.88</td>
<td>1.44</td>
<td>1.71</td>
</tr>
<tr>
<td>1997</td>
<td>4.50</td>
<td>3.03</td>
<td>2.29</td>
<td>1.90</td>
</tr>
<tr>
<td>1998</td>
<td>4.18</td>
<td>3.32</td>
<td>1.47</td>
<td>2.00</td>
</tr>
<tr>
<td>1999</td>
<td>4.45</td>
<td>3.52</td>
<td>1.52</td>
<td>2.17</td>
</tr>
<tr>
<td>2000</td>
<td>3.66</td>
<td>3.52</td>
<td>2.55</td>
<td>0.74</td>
</tr>
<tr>
<td>2001</td>
<td>0.75</td>
<td>2.93</td>
<td>0.00</td>
<td>-0.29</td>
</tr>
<tr>
<td>2002</td>
<td>1.60</td>
<td>2.62</td>
<td>-0.29</td>
<td>0.85</td>
</tr>
<tr>
<td>2003</td>
<td>2.51</td>
<td>2.62</td>
<td>0.88</td>
<td>0.99</td>
</tr>
<tr>
<td>2004</td>
<td>3.91</td>
<td>2.71</td>
<td>1.09</td>
<td>2.21</td>
</tr>
<tr>
<td>2005</td>
<td>3.22</td>
<td>2.66</td>
<td>1.80</td>
<td>1.09</td>
</tr>
</tbody>
</table>
To compare the contributions to growth, we need to compare the magnitudes,
$0.36(\Delta \bar{K}/K)$, $0.64(\Delta \bar{N}/N)$, and $\Delta \bar{Z}$. These values are presented in the table below.

<table>
<thead>
<tr>
<th>Year</th>
<th>$0.36(\Delta \bar{K}/K)$ (%)</th>
<th>$0.64(\Delta \bar{N}/N)$ (%)</th>
<th>$\Delta \bar{Z}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1996</td>
<td>1.04</td>
<td>0.92</td>
<td>1.71</td>
</tr>
<tr>
<td>1997</td>
<td>1.09</td>
<td>1.46</td>
<td>1.90</td>
</tr>
<tr>
<td>1998</td>
<td>1.20</td>
<td>0.94</td>
<td>2.00</td>
</tr>
<tr>
<td>1999</td>
<td>1.27</td>
<td>0.97</td>
<td>2.17</td>
</tr>
<tr>
<td>2000</td>
<td>1.27</td>
<td>1.63</td>
<td>0.74</td>
</tr>
<tr>
<td>2001</td>
<td>1.05</td>
<td>0.00</td>
<td>-0.29</td>
</tr>
<tr>
<td>2002</td>
<td>0.94</td>
<td>-0.19</td>
<td>0.85</td>
</tr>
<tr>
<td>2003</td>
<td>0.94</td>
<td>0.56</td>
<td>0.99</td>
</tr>
<tr>
<td>2004</td>
<td>0.98</td>
<td>0.70</td>
<td>2.21</td>
</tr>
<tr>
<td>2005</td>
<td>0.96</td>
<td>1.15</td>
<td>1.09</td>
</tr>
</tbody>
</table>

Most often, when output is growing, the biggest contribution to growth comes from increases in total factor productivity. In 1991 and in 2001, both bad years for growth, total factor productivity decreased. In the other years, growth in total factor productivity is usually the largest contributor to growth, while increases in capital and labor equally share the role of the leading cause of growth in the other years. In the later years, capital growth has come to be relatively more important than in the early years.